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Non-Integration, Integration, and the Decentralized Firm: Management of "Externalities vs. Private Benefits"

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Abstract

Based on the view of a firm as an entity of "units and activities", we study the optimal scope of a firm and the assignment of different types of managers to different types of firms and activities in a concrete IO setting with R&D and production activities. "Non-integrated firms" fail to consider the external effects that managers' decisions, especially R&D decisions have on other firms. While an "integrated firm" internalizes these externalities, it does not take into consideration the "private benefits" of managers. So, we consider a third candidate, the "Decentralized Firm", as a balance between the internalization of externality effects and the consideration of private benefits. We see that this third regime identifies the optimal delegation of authority inside a firm. Last, we compare the three regimes (organizational forms) from the viewpoint of "marginal incentives", and uncover some interesting economic implications.

Key Words Non-Integration, Integration, Decentralized Firm, Private Benefit, Internalization of Externalities.

JEL classification D23, L22.

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1. Introduction

1.1. The Importance of our Research: Real World Examples

We shall start by introducing some anecdotes based on U.S. and Japanese firms[1] which have motivated this paper.

Prior to World War 1, Sears Roebuck was a highly centralized mail-order catalog business dealing in "hard goods," such as tools. After the war, it introduced retail stores on the main streets of U.S. cities and expanded its product line to include "soft goods", such as clothing. Sears initially tried to maintain its centralized organization. However, consumer demands varied widely across the country, such that during the winter, warm coats would sell in the cold northern states but not in Florida or Texas. If the organization was to be able to respond to local conditions, managers in the different regional markets needed to be given more authority to run their local operations. Ultimately, Sears reorganized its business by establishing divisions on a regional basis, the managers of which had greater decision-making authority.

In the early1920s, before Sloan's reorganization of General Motors, the manufacturing managers of Buick, Cadillac, Chevrolet, Oakland, and Olds operated with sufficient independence but not with sufficient coordination among themselves or with the sales team. The central office poorly coordinated the plans and decisions of the independent managers. The separately chosen product strategies of the units lead to more competition among themselves than with Ford. Failure to coordinate on design standards also prevented the divisions from taking full advantage of the potential economies of scale when making or purchasing common components, like sparkplugs and bearings. There was also a failure to coordinate sales and production also existed. In summary, the former organization of *General Motors* was essentially a collection of car companies and suppliers operating without any centralized coordination, and was no model either. The multidivisional structure GM implemented resolved these problems because, in addition to its decentralized divisions, it had a central office with a strong, professional staff to plan strategy and coordinate divisional activities. At General Motors, each separated division made and sold a car targeted at an assigned market segment. Each had its own managerial team with <u>authority to make its own operating decisions</u>. Unlike other business organizations, GM's central office was not responsible for day-to-day operations, its primary role was to plan and coordinate overall strategy. It was also responsible for the coordination of the research and development functions of the corporation, and hosted group meetings to share ideas about how to improve products and reduce the manufacturing costs of each division. In summary, the structure of General Motors was changed by the creation of an organizational structure consisting of separate divisions matched with a central office that coordinated business strategies. Its great success led others to mimic GM's structure and strategy.

Hoshi-Okazaki (2001) explored the historical factors in the banking industry which brought about the 'Heisei bubble' and its recent collapse rash. The author points out that the decentralized organization of Japanese banks (e.g., Sumitomo and Mitsubishi) in the1970s and 80s led to an oversupply of liquidity, which was an important factor in the 'Heisei bubble' and its crash. On the other hand, there is a recent and increasing trend for

[1] The explanation for the U.S. cases is based on Milgrom and Roberts (1992).

big banks to merge (e.g., Sumitomo and Sakura). How and in which framework do we understand these phenomena?

1.2. Previous Literature and Our Paper

Do firm boundaries affect the allocation of resources? What determines where firm boundaries are drawn? Since the famous article (1937) by Ronald Coase, "The Nature of the Firm", these questions have already received a lot of research attention, as mentioned below. In contrast, we know relatively little about how these boundaries affect firm behavior. Empirical evidence seems to show that integrated firms do in fact behave quite differently from non-integrated ones. Also, how the organizational structure of the firm (e.g. decentralization and authority delegation) affects firm behavior is an important problem, as the historical cases showed. How do we formalize this into a model?

In order to understand the essence of these phenomena, we construct a simple, concrete IO model, with R&D and production activities, which is based on the ideas from the "Firm Scope" paper by Hart and Holmstrom (2002).

The existing literature on firms, based on incomplete contracts and property rights, which started with Grossman-Hart (1986) and Hart-Moore (1990), emphasizes that the ownership of assets--- and thereby firm boundaries--- is determined in such a way as to encourage ex-ante relation-specific investments by appropriate parties. Indeed, Hart-Moore (1990) focuses on ex ante relation-specific investments, by suppressing ex-post activities and assuming the ex-post reduced form value functions. It is also generally accepted that the Grossman-Hart-Moore approach applies to owner-manager firms better than large corporations.[2]

In this paper, we incorporate three important elements, in order to improve the existing modeling. First, we focus more on "ex-post decisions". They are non-contractible, but transferable through ownership. This is similar to the "ex-post decisions" view in Grossman-Hart (1986), except that they are "contractible" in Grossman-Hart (1986), while they are "non-contractible" in Hart-Holmstrom (2002) and our model, and so must be modeled in a *self-enforcing* way. Second, managers enjoy private benefits that are non-transferable. The importance of this idea seems to originate in Aghion-Bolton (1992). Since this concept of "private benefit" is an important element for analyzing the problem of how firm boundaries and the organizational structure of the firm (e.g., decentralization and authority delegation) affect firm behaviors, we also incorporate it into this model. Third, the decisions of managers will depend on managers' preferences, and different managers will typically have different preferences. Moreover, their preferences may depend on the scope of the firm they run. The implication of this assumption is that firm boundaries do matter: a merger between two firms will not be neutral, since the new manager of the integrated firm will not have, and in general cannot have, the same preferences as the two previous managers.

By employing a concrete IO model with these ingredients, we can study the optimal scope of a firm, and the assignment of different types of managers to different types of firms and activities. We first find that "Non-integrated firms" fail to account for the external effects that their decisions, especially R&D decisions have on other firms, while an "integrated firm" can internalize such externalities, but it does not take into

[2] For a critique, e.g., see Holmstrom-Roberts (1998).

consideration the private benefits of managers. This is a basic trade off between the two organizational forms. Next, we consider a third candidate, the "Decentralized Firm", which can be viewed as an intermediate form between the "Non-Integration" and the "Integration" regimes. The idea is that certain decisions should be put in the hands of someone with different preferences from the managers in order to create a balance between the internalization of externality effects and the consideration of private benefits. We see that this framework identifies the optimal delegation of (formal) authority inside a firm. Then, we compare the equilibrium incentives in these three second-best regimes, "Integrated", "Non-Integrated" and "Decentralized", with those of the first best regime, the joint surplus maximization regime, for two units. We then clarify in which direction a distortion arises to get the economic implications of the three regimes.

2. "Non-Integration": Regime 1

In this regime, the game is played by a pair of independent firms indexed by $i = 1,2$. Each firm allocates resources for the production of goods and for investment in technological knowledge. The firms are assumed to have an inverse market demand function defined by;

$$p_i\left(q_i, q_j\right) = A - \alpha q_i - \beta q_j, \text{ with } \alpha \geq 0 \qquad (1)$$

Each firm operates in a market which is represented by an inverse demand function relating the average price of firm i to its own supply q_i and the supply of firm $j: q_j$. The parameter β is an indicator of complementarity or substitutability between the production activities of the two firms. A negative β is an indicator of substitute goods, and identifies the degree of substitutability between the two firms.

Also, the firms are assumed to have cost reducing innovation with a spillover rate ϕ, which is given by the following total cost formula[3];

$$c_i\left(e_i, e_j; q_i\right) = \frac{1}{e_i + \phi e_j} q_i^2, 0 \leq \phi \leq 1, \quad i = 1,2 \qquad (2)$$

In this economy, each firm is facing a total cost c_i depending on the level of production activity. However, the marginal cost of production activity is negatively related to both the innovation level of the firm and the innovation of the other firm. Hence, the cost specification above implies that the cost structures of different firms producing different goods are interdependent. The effect of cost reducing technology on the formula above implies that, because of technological externalities, the prevailing marginal cost in a given firm (unit) depends on its own R&D as well as the R&D of the other firm, whether

[3]We incorporate the cost reducing innovation/R&D activity e into the model, because we want to analyze the problem of "delegation of authority" as well. As the reader will see later, the production decision is delegated to the local managers in the "decentralized firm" regime, while R&D activity e is determined by the professional manager (e.g., the general office) with a different preference from the local managers.

they are a direct competitor such as in case of differentiated product, or a complementary firm (unit). As a consequence, R&D plays a different role here than in the case of homogeneous products since externalities might benefit the innovating firm by lowering the costs of the complement firm. Indeed, in lowering its own marginal costs, a given firm may lower the marginal cost of the other firm, with no direct negative strategic impact on the firm. The impact of innovation on the cost structure of the firm depends on the magnitude of the spillover parameter ϕ. The rate of externality ϕ measures the degree of appropriability of the innovation outcome. If ϕ is close to zero, this corresponds to economies with a high degree of R&D output appropriability, and consequently a high incentive to conduct R&D. The other polar case corresponds to situations where ϕ is close to one. This is interpreted as a low degree of appropriability and hence a low private incentive to innovate. For any intermediary value, the economy is characterized by imperfect spillovers.

Each firm's strategy consists of the pair $\sigma_i : (e_i, q_i), i = 1,2$ composed of an innovation effort and an amount of production such that $\sigma_i \in \Gamma_i$ where Γ_i denotes the strategy space of firm i.

The monetary profit function R_i of firm i depends on the production and R&D strategies such that

$$R_i(\sigma_i, \sigma_j) = p_i q_i - \frac{1}{e_i + \phi e_j} q_i^2 - e_i \qquad i, j = 1,2 \tag{3}$$

In addition, we assume that each firm (in particular, its manager) enjoys a "private benefit" from its production activities in the market.[4] The benefit is non-monetary and non-transferable, such as job satisfaction and pride/self esteem generated from producing more. We formulate this "private benefit" in a simple way such that

$$w_i(q_i) = w \cdot q_i, \text{ with } w \geq 0, \quad i = 1,2 \tag{4}$$

In the "Non-Integration" regime, each firm (or its manager) maximizes the sum of the monetary profit and the private benefit

$$V_i = R_i + w_i = (p_i + w) q_i - \frac{1}{e_i + \phi e_j} q_i^2 - e_i \tag{5}$$

This is an important assumption.

The Nash equilibrium for this noncooperative game is a pair of Nash equilibrium strategies

$$V_i(\sigma_i^*, \sigma_j^*) \geq V_i(\sigma_i, \sigma_j^*), \forall \sigma_i \in \Gamma_i; i, j = 1,2 \tag{6}$$

[4] Recent literature incorporates this concept into models. E.g., see Aghion-Bolton (1992) and Hart-Holmstrom (2002). Also see the survey of the points by Dewatripont (2001).

Therefore, a Nash equilibrium of this innovation-production game is a pair of actions (σ_i, σ_j) such that no firm (or its manager) has an incentive to deviate from this choice taking the other firm choice as given. The Nash equilibrium is a simultaneous maximization of the sum of the monetary profit and the private benefit for each firm i. Simultaneity implies that each firm has not yet observed the other firm's R&D and production levels, when choosing its own, therefore a firm is assumed to anticipate them correctly. Hence, the Nash equilibrium of the game must satisfy the following first order conditions,

$$\frac{\partial V_i}{\partial q_i} = \frac{\partial(R_i + w_i)}{\partial q_i} = 0, \quad i,j = 1,2 \tag{7.1}$$

and

$$\frac{\partial V_i}{\partial e_i} = \frac{\partial(R_i + w_i)}{\partial e_i} = 0, \quad i,j = 1,2 \tag{7.2}$$

which yields the reaction function of each firm given by;

$$e_i = \frac{A + w - 2}{\phi\beta + 2\alpha} - \frac{\beta + 2\alpha\phi}{\phi\beta + 2\alpha} e_j, \quad i,j = 1,2 \tag{8}$$

Under the above conditions, we have the following propositions[5];

Proposition1

a) There exists a unique symmetric equilibrium innovation level with spillover ϕ given by:

$$e^{NI} = \frac{A + w - 2}{(1+\phi)(2\alpha + \beta)} \tag{9}$$

b) The corresponding unique symmetric Nash equilibrium production level is;

$$q^{NI} = \frac{A + w - 2}{(2\alpha + \beta)} \tag{10}$$

Based on (9) and (10), $q^{NI} = \frac{1}{(1+\phi)} e^{NI}$.

c) $\frac{\partial e^{NI}(\phi, w, \alpha, \beta)}{\partial \phi} < 0$; an increase in the external effects lowers the equilibrium incentive to innovate. This is a "Free Rider effect".

[5] For detailed calculations, see Appendix 1.

d) $\frac{\partial e^{NI}(\phi,w,\alpha,\beta)}{\partial w} > 0$; larger "private benefits" result in higher equilibrium incentives to innovate.

e) $\frac{\partial q^{NI}(w,\alpha,\beta)}{\partial w} > 0$; larger "private benefits" result in higher equilibrium incentives to produce.

f) $\frac{\partial e^{NI}(\phi,w,\alpha,\beta)}{\partial \beta} < 0$; the incentive to innovate is lower, if the market share of the substitute firm increases. Conversely, in the case of complementary production activity, the incentive to innovate increases whenever the complement firm's demand increases.

Interpretation

The existence of technological externalities has a negative impact on each independent firm's innovation effort. That is, the existence of spillovers reduces the private incentives to invest in technological knowledge, due to the free rider effect, as it may benefit from the innovation of the other firm.

Proposition d) and e) reveals that the existence of private benefit w has a positive impact on each firm/unit's production decision and innovation effort. This occurs because the increase of private benefit w increases the marginal benefit of increasing $q_i, i=1,2$, that is, it makes the reaction function of i in q_i shift outward. Thus, the equilibrium quantity level q^{NI} increases with w. This increases the marginal revenue of innovation e_i for each firm/unit, and also increases the equilibrium effort level e^{NI}.

All other things being equal, an increase in the inverse market demand elasticity α tends to reduce the incentive to innovate and produce. If α increases, it means that the marginal demand declines and this lowers the production level. Hence a smaller market size results in a lower incentive to innovate.

Proposition f) reveals that the increase (decrease) in the demand for the complementary firm increases (reduces) the innovation efforts of the firm i. In contrast, the increase in the market share of a rival firm j (i.e., a substitute firm) decreases the innovation effort of firm i. Therefore, as intuition suggests, the effect on the innovation effort does depend on changes in the condition of the other firm. Such influence depends on the nature of inter-firm (unit) relationship i.e., whether the goods of the firms are *complements or substitutes.*

3. "Integration"[6]: Regime 2

In this regime, the two firms are integrated with each other, and a "professional" manager, who maximizes the joint monetary profits of the two firms (units) by running

[6] "Integration" corresponds to "Centralization" or the "Centralized Firm" in the anecdotes in the introduction.

the integrated firm.[7] The important point is that the manager does not consider the "private benefits" of the manager does not consider the "private benefits" of the two original firm managers.[8] In this game, she decides a joint R&D effort level and separate production levels that maximize the joint monetary profit as a function of q_i, q_j and e. The financing of the R&D is borne by the two divisions (units) according to a prior sharing rule $(\theta_1, \theta_2) = (1/2, 1/2)$. Therefore, $e_i = (1/2)e$, $i = 1, 2$. We assume that the professional manager decides a total R&D level e and commands the execution of the half of e to each unit. We rule out the problem of *enforceability*, i.e., each unit follows the command to execute $(1/2)e$. This corresponds to the situation, where, in centralized organizations, top management and its staff can control the operating decisions of their units (divisions) directly.

The equilibrium for this game is a triplet (e, q_i, q_j) chosen in such a way that the "professional manager" maximizes the joint monetary profit. The joint monetary profit function is defined according to the following;

$$R^I(e, q_i, q_j) = R_i(e, q_i) + R_j(e, q_j) \tag{11}$$

which is equivalent to:

$$R^I(e, q_i, q_j) = \sum p_i q_i - \sum \frac{1}{e_i + \phi e_j} q_i^2 - \sum e_i \tag{12}$$

and hence

$$R^I(e, q_i, q_j) = \sum p_i q_i - \sum \frac{1}{(1+\phi)e} q_i^2 - e \tag{13}$$

The first order conditions of the joint monetary profit maximization are given by the following system of equations;

$$\frac{\partial R^I(e, q_i, q_j)}{\partial q_i} = 0, \quad i, j = 1, 2 \tag{14.1}$$

and

$$\frac{\partial R^I(e, q_i, q_j)}{\partial e} = 0 \tag{14.2}$$

which yields the following propositions[9];

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[7] If the manager of the firm 1 becomes a new manager of the integrated firm, he/she will maximize the joint monetary profits R^I plus his/her private benefit $w_1(q_1) = wq_1$, that is, $R^I + w_1$. For simplicity, we assume that a new professional manager will be employed from outside, and so he/she has no linkage with the original firms 1 and 2.

[8] Hence, this is not merely a "cooperative game" between the two firms, in the game theoretical sense.

[9] For detailed calculations, see Appendix 2.

Proposition2

a) The cooperative R&D effort is given by;

$$e^I = \frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A - \frac{2}{\sqrt{1+\phi}}\right] \tag{15}$$

b) The corresponding symmetric Nash equilibrium aggregate production level is as follows;

$$q^I = q_1^{\ I} + q_2^{\ I} = \frac{1}{(\alpha+\beta)}\left[A - \frac{2}{\sqrt{1+\phi}}\right] \tag{16}$$

We see from (15) and (16) that $e^I = \frac{1}{\sqrt{1+\phi}} q^I$ (17)

c) Comparison between "Non-Integration" vs. "Integration"; when $\beta \geq 0$, i.e., there exists a competing relationship (substitutability) between the two units, as the degree of private benefit is greater, and as the spillover rate (the degree of externality) is smaller, q^{NI} tends to be greater than $\overline{q}^I$ (the average production level under "Integration")

Proof

$$q^{NI} = \frac{A+w-2}{2\alpha+\beta} \geq \frac{1}{2(\alpha+\beta)}\left[A - \frac{2}{\sqrt{1+\phi}}\right] = \overline{q}^I$$

$$\Leftrightarrow \frac{2(\alpha+\beta)}{2\alpha+\beta} \frac{[A+w-2]}{\left[A - \frac{2}{\sqrt{1+\phi}}\right]} \geq 1$$

When $\beta \geq 0$, a sufficient condition for the above condition is $w \geq 2\left(1 - \frac{1}{\sqrt{1+\phi}}\right)$.

As can be seen, when the degree of private benefit w is greater, and as the spillover rate (the degree of externality) ϕ is smaller, this inequality tends to be satisfied. That is, q^{NI} tends to be greater than $\overline{q}^I$. This reflects the basic trade-off, which lies between "Non-Integration (consideration of w and ignorance of ϕ) vs. Integration (ignorance of w and internalization of ϕ)"

4. "Decentralized Firm"[10]: Regime 3

The basic trade-off so far is as follows. In the "Non-Integration" regime, the managers of the two independent firms do not consider the positive externalities of their R&D efforts, and thus it will generally lead to the underinvestment in R&D, though the 'private benefit' effect wq mitigates this underinvestment effect to some degree. On the other hand, in the "Integration" regime, a "professional manager" maximizes the sum of the monetary profits, neglecting "private benefits", such as job satisfaction and pride/self esteem of the local managers. This reduces the joint total surplus. Therefore, there exists a trade-off between the internalization of externalities from R&D efforts and the loss of the private benefits in the comparison of two regimes presented so far. So now, we shall consider a third candidate, the "Decentralized Firm", which can be viewed as an intermediate form between the "Non-Integration" and "Integration" regime.

As stated by Milgrom and Roberts (1992), historically speaking, the multidivisional firm (**"Decentralized Firm"** in our model) emerged in an era when there were only two major alternatives: highly centralized organizations (in our model, **"Integrated"** firms), such as that which had previously existed at Sears and du Pont, and organizations with almost no control (that is, a "too decentralized" form, represented in our model, by **"Non-Integrated"** firms), such as the form that existed at General Motors before the reforms undertaken by Alfred Sloan. The reason for the multi-divisional structure ("Decentralized Firm") in General Motors was to carry out Sloan's new market-segmentation strategy, which also fits our "two market" model. General Motors then placed product and marketing decisions in the hands of divisional managers, i.e., **authority was delegated** to the lower levels of the organization.

Based on this evidence, we hypothesize that in the "Decentralized Firm" regime, a professional manager chooses a R&D effort level with the objective of maximizing the joint monetary profits[11], while the local managers of the two units (divisions) maximizes the sum of their unit's monetary profit and their own "private benefit", independently and simultaneously. They are delegated the (formal) authority of choosing the production activities.

Now, let us show how the model works.[12] First, each of the local managers of the two units chooses the production level q_i in order to maximize the sum of his monetary profit

[10] The "Decentralized Firm" regime corresponds to the "M-form structure" ala Williamson (1985) or the "Multidivisional form" (Milgrom and Roberts (1992)), though we do not deal with the "information asymmetry" and "internal capital market "argument as their models do. Suzuki (2002) examines such problems in a principal-supervisor-two agent hierarchy. This paper is more interested in the optimal scope of a firm i.e., the determination of "firm boundary" through the trade off between the internalization of externalities and the consideration of "private benefit".

[11] This may be said to reflect the idea by Williamson (1985, pp283) that the "M-form structure" removes the general office executives, i.e., the professional manager in our model, from *partisan involvement* in the functional parts and assigns operating responsibilities to the divisions, and that the general staff, moreover, is supported by an elite staff, independent of the divisional interests.

[12] Carmichael (1983) considers a model, where the principal also makes a common productive effort and the two agents compete with each other under tournament or relative performance evaluation. Their efforts are chosen in a self-enforcing way. It is a bit similar to our model in its structure and solution, but does not consider the problems of "firm boundary" and "delegation of authority".

and his private benefit, given that the "professional manager" decides the common R&D level e. So, manager i maximizes with respect to q_i, given e and q_j;

$$V_i^{DF} = R_i^{DF} + w_i = (p_i + w) q_i - \frac{1}{(1/2)(1+\phi)e} q_i^2 - \frac{1}{2} e, \quad i, j = 1, 2 \tag{18}$$

That is, the following first order conditions must be satisfied.

$$\frac{\partial V_i^{DF}}{\partial q_i} = \frac{\partial \left(R_i^{DF} + w_i \right)}{\partial q_i} = 0, \;\; i, j = 1, 2 \tag{19}$$

which is equivalent to these set of equations

$$\left(A + w - \alpha q_i - \beta q_j \right) - \alpha q_i - \frac{2}{(1/2)(1+\phi)e} q_i = 0, \quad i = 1, 2 \tag{20}$$

We restrict our attention to the symmetric equilibrium $q_i = q_j = q^{DF}$, and thus we have;

$$A + w - (\alpha + \beta) q^{DF} - \alpha q^{DF} - \frac{2}{(1/2)(1+\phi)e} q^{DF} = 0$$

$$\Leftrightarrow A + w = \left[(2\alpha + \beta) + \frac{4}{(1+\phi)e} \right] q^{DF} \tag{21}$$

Next, the "professional manager" maximizes the joint monetary profit with respect to e, given that the two local managers choose the symmetric Nash equilibrium production level q^{DF}. Hence, the professional manager's objective is to maximize with respect to e given q^{DF}

$$\begin{aligned} R^I \left(e, q^{DF} \right) &= R_i \left(e, q^{DF} \right) + R_j \left(e, q^{DF} \right) \\ &= \sum p_i \cdot q^{DF} - \frac{2}{(1/2)(1+\phi)e} \left(q^{DF} \right)^2 - e \end{aligned} \tag{22}$$

The first order condition is;

$$\frac{2}{(1/2)(1+\phi)e^2} \left(q^{DF} \right)^2 = 1 \tag{23}$$

The only economically admissible root is;

$$q^{DF} = \frac{\sqrt{1+\phi}}{2} e \;\; \Leftrightarrow e = \frac{2}{\sqrt{1+\phi}} q^{DF} \tag{24}$$

Substituting (24) into (21), we have;

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \tag{25}$$

and

$$e^{DF} = \frac{2}{\sqrt{1+\phi}} q^{DF} = \frac{2}{\sqrt{1+\phi}} \cdot \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \tag{26}$$

Therefore, we have the following proposition;

Proposition 3

a) The symmetric Nash equilibrium production level is;

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \tag{27}$$

b) The corresponding equilibrium R&D effort is given by;

$$e^{DF} = \frac{2}{\sqrt{1+\phi}} q^{DF} = \frac{2}{\sqrt{1+\phi}} \cdot \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \tag{28}$$

c) We have that; for all $0 \le \phi \le 1, w \ge 0$

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \ge \frac{1}{2\alpha+\beta}[A + w - 2] = q^{NI} \tag{29}$$

and

$$e^{DF} = \frac{2}{\sqrt{1+\phi}} q^{DF} \ge \frac{1}{1+\phi} q^{NI} = e^{NI} \tag{30}$$

The "Decentralized Firm" internalizes the externality effect of R&D, and so under the lower production cost, generates the incentive to produce more, due to the private benefits. Hence, we can say that the "Decentralized Firm" manages the trade-off between externality vs. private benefit more successfully than "Non-Integration".

d) Comparison between the "Decentralized Firm"and "Integration"; the condition for $q^{DF} \ge \overline{q}^{I}$ is as follows.

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A + w - \frac{2}{\sqrt{1+\phi}}\right] \ge \frac{1}{2(\alpha+\beta)}\left[A - \frac{2}{\sqrt{1+\phi}}\right] = \overline{q}^{I}$$

$$\Leftrightarrow w \geq \frac{-\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right] \quad (31)$$

d.1) when $\beta \geq 0$, i.e., there exists a competing relationship (substitutability) between the two units, we always have $q^{DF} \geq \bar{q}^I$.

d.2) when $\beta \leq 0$, i.e., when there exists a complementary relation between the two units, as the amount of private benefit w is greater, and as the spillover rate (the degree of externality) ϕ is smaller, this inequality tends to be satisfied. In contrast, as w is smaller, the degree of complementarity $|\beta|$ is greater, so is the incentive to expand production through the internalization of *positive externalities* of q under "Integration".

5. "Joint Surplus Maximization" regime

In this section, we analyze the "Joint Surplus Maximization" regime, which is the first best optimum regime for the two units. This is a "bench mark" regime, since there does not exist any manager whose preference is to maximize the joint surplus of the two units. The manager of each unit, if he/she becomes a top manager, will maximize the joint monetary profits R^I plus his/her private benefit $w_1(q_1)=wq_1$, that is, R^I+w_1. The professional manager, who is employed from outside, and has no linkage with the original units, will maximize only the joint monetary profits R^I . In other words, no manager exists, whose 'vision' is to maximize the joint surplus of the two units.[13] That's the reason why the "Joint Surplus Maximization" cannot be implemented in equilibrium.

The joint surplus function is expressed as the sum of the monetary profit function and the private benefit one, and is given by;

$$\Pi^{JS}=R^I\left(e,q_i,q_j\right)+w_i\left(q_i\right)+w_j\left(q_j\right)$$

which is equivalent to the following equation;

$$R_i\left(e,q_i\right)+R_j\left(e,q_j\right)+w\cdot\left(q_i+q_j\right)=\sum\left(p_i+w\right)q_i-\sum\frac{1}{(1/2)(1+\phi)e}q_i^2-e$$

The first order conditions of the joint surplus maximization are given by;

$$\frac{\partial\Pi^{JS}\left(e,q_i,q_j\right)}{\partial q_i}=0,\quad i,j=1,2$$

and

$$\frac{\partial\Pi^{JS}\left(e,q_i,q_j\right)}{\partial e}=0$$

So we have the following results[14].

[13] See, Hart and Holmstrom (2002).

[14] As for the detailed calculations, see Appendix 3.

The "first best" R&D innovation effort is given by;

$$e^{JS} = \frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$$

The corresponding symmetric Nash equilibrium aggregate production level is

$$q^{JS} = \frac{1}{(\alpha+\beta)}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$$

6. Summary of the Results and Implications

Now, let us summarize the result so far.

	Production q	R&D innovation e
Non-Integration	$q^{NI} = \frac{A+w-2}{(2\alpha+\beta)}$	$e^{NI} = \frac{A+w-2}{(1+\phi)(2\alpha+\beta)}$
Integration	$\bar{q}^{I} = \frac{1}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right]$	$e^{I} = \frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A-\frac{2}{\sqrt{1+\phi}}\right]$
Decentralized Firm	$q^{DF} = \frac{1}{2\alpha+\beta}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$	$e^{DF} = \frac{2}{(2\alpha+\beta)\sqrt{1+\phi}}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$
Joint Surplus Maximizatio n	$q^{JS} = \frac{1}{2(\alpha+\beta)}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$	$e^{JS} = \frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right]$

Table 1

We have the following proposition.

<u>Proposition4</u>

1) If $\beta \geq 0$ (substitutable goods) and $w \geq 2\left(1-\frac{1}{\sqrt{1+\phi}}\right)$, then we have $q^{DF} \geq q^{NI} \geq \bar{q}^{I}$.

2) We have $q^{NI} \leq q^{DF} \leq \bar{q}^{I}$, if and only if $\beta \leq 0$ (complementary goods) and
$0 \leq w \leq \frac{-\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right]$.

3) We have $q^{JS} \geq \bar{q}^{I}$ and $e^{JS} \geq e^{I}$, for $w \geq 0$ and $0 \leq \phi \leq 1$

4) As ϕ goes to 0, q^{NI} tends to be greater than q^{JS} for $\beta > 0$. As β (>0) goes to 0, q^{JS} tends to be greater than q^{NI} for $0 \leq \phi \leq 1$.

5) Suppose $\phi = 0$ (or small enough). If $\beta > 0$ and $w > 0$, we have $q^{NI} > q^{JS} > q^{I}$.

6) If $\beta \geq 0$, then we have $q^{DF} \geq q^{JS}$ and $e^{DF} \geq e^{JS}$, and vice versa.

Proof

1) The comparison of productions between "Non-Integration" and "Integration" is;

$$q^{NI} = \frac{A+w-2}{2\alpha+\beta} \geq \frac{1}{2(\alpha+\beta)}\left[A - \frac{2}{\sqrt{1+\phi}}\right] = \overline{q}^{I}$$

$$\Leftrightarrow \frac{2(\alpha+\beta)}{2\alpha+\beta} \frac{[A+w-2]}{\left[A-\frac{2}{\sqrt{1+\phi}}\right]} \geq 1$$

When $\beta \geq 0$, a sufficient condition for the above condition is $w \geq 2\left(1-\frac{1}{\sqrt{1+\phi}}\right)$.

We also know that for all $0 \leq \phi \leq 1, w \geq 0$

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right] \geq \frac{1}{2\alpha+\beta}[A+w-2] = q^{NI}$$

Hence, if $\beta \geq 0$ and $w \geq 2\left(1-\frac{1}{\sqrt{1+\phi}}\right)$, then we have $q^{DF} \geq q^{NI} \geq \overline{q}^{I}$.

2) Next, we consider the $\beta \leq 0$ case. The comparison of productions is;

$$q^{DF} = \frac{1}{2\alpha+\beta}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right] \leq \frac{1}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right] = \overline{q}^{I}$$

$$\Leftrightarrow w \leq \frac{-\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right]$$

and

$$q^{NI} = \frac{A+w-2}{2\alpha+\beta} \leq \frac{1}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right] = \overline{q}^{I}$$

$$\Leftrightarrow w \leq \frac{2\alpha+\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right] - (A-2)$$

Then, we find that;

$$\frac{-\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right]\leq\frac{2\alpha+\beta}{2(\alpha+\beta)}\left[A-\frac{2}{\sqrt{1+\phi}}\right]-(A-2)$$
$$\Leftrightarrow A-2\leq A-\frac{2}{\sqrt{1+\phi}},\quad \text{for } 0\leq\phi\leq 1$$

Therefore, if $q^{DF}\leq\overline{q}^{I}$ holds, then we always have $q^{NI}\leq\overline{q}^{I}$. Further we already have $q^{DF}\geq q^{NI}$. Hence, we will have $q^{NI}\leq q^{DF}\leq\overline{q}^{I}$. And, the condition $q^{DF}\leq\overline{q}^{I}$ tends to be satisfied as w is smaller, and as ϕ is close to 1.

3) This is obvious from the table. The intuition is that in the joint surplus maximization regime the effect of the private benefit $w\cdot q$ is internalized, while in the "Non-Integration" regime it is neglected.

4) This is due to the fact that $\frac{q^{JS}}{q^{NI}}=\frac{2\alpha+\beta}{2(\alpha+\beta)}\cdot\frac{A+w-2/\sqrt{1+\phi}}{A+w-2}$.

The intuition is that in the joint surplus maximization regime, the externality effect of production q is internalized, while it is not considered in the "Non-Integration" regime. It brings about a direction to more output expansion in the "Non-Integration" regime. But, the spillover effect ϕ is internalized *only* in the joint surplus maximization regime. This leads to more output expansion through greater cost reductions.

5) When $\phi=0$, we have

$$q^{NI}=\frac{A+w-2}{(2\alpha+\beta)},\overline{q}^{I}=\frac{1}{2(\alpha+\beta)}[A-2],\text{ and } q^{JS}=\frac{1}{2(\alpha+\beta)}[A+w-2]$$

This gives the above result. $q^{NI}>q^{JS}$ comes from the internalization of the negative externality on production q, and $q^{JS}>q^{I}$ is due to the fact that in the "Integration" regime private benefits $w\cdot q_i, i=1,2$ are ignored by the top manager.

6) From the result of the table, we find that the ratios between q^{DF} and q^{JS}, and between e^{DF} and e^{JS} are equal to

$$\frac{q^{DF}}{q^{JS}}=\frac{e^{DF}}{e^{JS}}=\frac{2(\alpha+\beta)}{2\alpha+\beta}\begin{cases}\geq 1 & \text{for } \beta\geq 0\\ \leq 1 & \text{for } \beta\leq 0\end{cases}$$

$\beta\geq 0$ indicates that the goods of the two units are substitutable. This can be easily understood based on a comparison between the Cournot-Nash behavior (in the 'Decentralized Firm' regime) and the internalization of externalities brought about by the productive activities (in the 'Joint Surplus Maximization' regime), we can easily understand this result. The reasoning is similar for $\beta\leq 0$ (complementary goods case).

Implication

We shall consider the implication of the results.

Case 1: $q^{DF} \geq q^{NI} \geq \bar{q}^{I}$ under the condition that $\beta \geq 0$ and $w \geq 2\left(1-\frac{1}{\sqrt{1+\phi}}\right)$ corresponds to the situation where the "decentralized firm" brings about the greatest expansion of production between the three original forms. It may explain, from a viewpoint of the organizational form, why the decentralization in the organizations of Japanese banks (e.g., Sumitomo and Mitsubishi) in 1970's and 80's led to the oversupply of liquidity.[15].

Case2: $q^{NI} \leq q^{DF} \leq \bar{q}^{I}$ with $\beta \leq 0$ corresponds to the case where the effect of the internalization of the externalities under the "Integration" regime is largest. In this case, the production expands most through both the large reduction in marginal costs (the internalization of the positive externality of cost reducing activity e) and the internalization of the positive externality of production q for $\beta \leq 0$. This might explain the recent tendency of big banks to merge (e.g., Sumitomo and Sakura).[16]

6.1. "Zero-sum" private benefit function: $w_1(q_1)+w_2(q_2)=0$

If we assume that the private benefit functions have a "zero-sum" structure, that is, $w_1(q_1)+w_2(q_2)=0$, the joint surplus maximization is equivalent to the joint monetary profits maximization. It is achieved in the "Integration", i.e., "Centralization" regime. Hence, the "Integration" regime is the (first-best) optimal, and we point out the following.

Corollary

If the private benefits are distributed *unequally*, such as in "Zero-Sum" structure, $w_1(q_1)+w_2(q_2)=0$, "Integration" or "Centralization" is optimal.

7. Conclusion

In this paper, a two-unit model of production and innovation is presented. The model presented is similar to the traditional view of the firm as a technologically defined entity that makes decisions about inputs, outputs, and investments. Holmstrom-Roberts (1998) points out the importance of viewing a firm as an entity of "units and activities", as well as considering "asset ownership": the core of the property right theory by Grossman-Hart-Moore.

Based on the view of Hart-Holmstrom (2002), which formalizes the view of the firm as an entity of "units and activities", our model studies, by employing a concrete IO setting,

[15]Hoshi-Okazaki (2001) points out this fact in their paper, which studies the historical factors in the banking industry that brought about the 'Heisei bubble' and its crash.

[16] This could explain why Coca-Cola Japan Company has recently decided to move from a "Decentralized Firm" to an "Integrated Firm"

the optimal scope of a firm, and the assignment of different types of managers to different types of firms and activities. We find that Non-integrated firms fail to account for the external effects that managers' decisions, especially R&D decisions have on other units. An integrated firm can internalize such externalities, but it does not take into consideration the private benefits of managers, which also play a critical role in our analysis. So, in order to create a balance between the internalization of externality effects and the consideration of the private benefits, we consider a third candidate, the "Decentralized Firm", which is an intermediate form between the "Non-Integration" and "Integration" regimes. It means that production decisions are put in the hands of the local managers, who have different preferences from the professional manager, who takes charge of R&D decision. We see that this framework identifies the optimal delegation of (formal) authority inside a firm. Last, we compared three regimes (organizational forms) from the viewpoint of "marginal incentives", and obtain some interesting economic implications.

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<u>Appendix 1</u> Derivation of the equilibrium outcome of the "Non-Integration' regime

In the "Non-Integration" regime, each firm (its manager) maximizes the <u>sum</u> of the monetary profit and the private benefit. It depends on the output strategies and the innovation strategies of each firm (manager);

$$V_i = R_i + w_i = (p_i + w)q_i - \frac{1}{e_i + \phi e_j} q_i^2 - e_i, \quad i = 1,2 \tag{1}$$

Substituting the inverse market demand of firm i into equation (1), we have;

$$\begin{aligned} R_i + w_i &= (p_i + w)q_i - \frac{1}{e_i + \phi e_j} q_i^2 - e_i \\ &= (A + w - \alpha q_i - \beta q_j)q_i - \frac{1}{e_i + \phi e_j} q_i^2 - e_i, \quad i = 1,2 \end{aligned} \tag{2}$$

The Nash equilibrium of this game must satisfy the following first order conditions;

$$\frac{\partial V_i}{\partial q_i} = \frac{\partial (R_i + w_i)}{\partial q_i} = 0, \quad i,j = 1,2 \tag{3.1}$$

and

$$\frac{\partial V_i}{\partial e_i} = \frac{\partial (R_i + w_i)}{\partial e_i} = 0, \quad i,j = 1,2 \tag{3.2}$$

which is equivalent to these set of equations;

$$(A + w - \alpha q_i - \beta q_j) - \alpha q_i - \frac{2}{e_i + \phi e_j} q_i = 0, \quad i,j = 1,2 \tag{4.1}$$

and

$$\frac{1}{(e_i + \phi e_j)^2} q_i^2 - 1 = 0, \quad i,j = 1,2 \tag{4.2}$$

hence the system becomes;

$$A + w - 2\alpha q_i - \beta q_j - \frac{2}{e_i + \phi e_j} q_i = 0, \quad i,j = 1,2 \tag{5.1}$$

and

$$\frac{1}{(e_i + \phi e_j)^2} q_i^2 - 1 = 0, \quad i,j = 1,2 \tag{5.2}$$

The solution for equation (5.2) implies both a negative and a positive root, but the only economically admissible root is;

$$q_i = e_i + \phi e_j, \quad i,j=1,2 \tag{6}$$

Substituting the value of q_i and q_j from (6) into (5.1) yields;

$$A+w-\beta\left(e_j+\phi e_i\right)-2\left(\alpha+\frac{1}{e_i+\phi e_j}\right)\left(e_i+\phi e_j\right)=0, \quad i,j=1,2$$

$$\Leftrightarrow A+w-\beta\left(e_j+\phi e_i\right)-2\left(\alpha\left(e_i+\phi e_j\right)+1\right)=0, \quad i,j=1,2 \tag{7}$$

We have the following equation set;

$$\left(A+w-2\right)-\left(\beta+2\alpha\phi\right)e_j-\left(\phi\beta+2\alpha\right)e_i=0, \quad i,j=1,2 \tag{8}$$

From (8) we have the reaction function of each player, given by the following relationship;

$$e_i=\frac{A+w-2}{\phi\beta+2\alpha}-\frac{\beta+2\alpha\phi}{\phi\beta+2\alpha}e_j, \quad i,j=1,2 \tag{9}$$

By solving the system of equations (9), we have the symmetric equilibrium innovation with spillover rate ϕ;

$$e^{NI}=\frac{A+w-2}{\left(1+\phi\right)\left(2\alpha+\beta\right)} \tag{10}$$

The corresponding unique symmetric Nash equilibrium production and equilibrium price levels are;

$$q^{NI}=\frac{A+w-2}{\left(2\alpha+\beta\right)}, \quad p^{*}=A-\left(\alpha+\beta\right)\frac{A+w-2}{\left(2\alpha+\beta\right)} \tag{11}$$

Appendix.2; Derivation of the equilibrium outcome of "Integration" regime

The joint monetary profit function is expressed as a function of q_i and q_j and e, and is given by;

$$R^{I}\left(e,q_i,q_j\right)=R_i\left(e,q_i\right)+R_j\left(e,q_j\right) \tag{12}$$

which is equivalent to the following equation;

$$R^{I}\left(e,q_i,q_j\right)=\sum p_i q_i-\sum\frac{1}{\left(1/2\right)\left(1+\phi\right)e}q_i^2-e \tag{13}$$

The first order conditions of the joint monetary profit maximization are given by;

$$\frac{\partial R^I\left(e,q_i,q_j\right)}{\partial q_i}=0,\quad i,j=1,2 \tag{14.1}$$

and

$$\frac{\partial R^I\left(e,q_i,q_j\right)}{\partial e}=0 \tag{14.2}$$

yielding the following set of equations;

$$\sum\partial\left(A-\alpha q_i-\beta q_j\right)q_i\big/\partial q_i-\frac{2}{(1/2)(1+\phi)e}q_i=0;i,j=1,2 \tag{15.1}$$

and;

$$\frac{1}{(1/2)(1+\phi)e^2}q_1^2+\frac{1}{(1/2)(1+\phi)e^2}q_2^2=1 \tag{15.2}$$

(15.1) and (15.2) reflects the "internalization of externalities" by $q_i, i=1,2$ and e. Hence, we have;

$$\partial\left(A-\alpha q_i-\beta q_j\right)q_i\big/\partial q_i+\partial\left(A-\alpha q_j-\beta q_i\right)q_j\big/\partial q_i-\frac{2}{(1/2)(1+\phi)e}q_i=0;i,j=1,2 \tag{16.1}$$

and;

$$\frac{1}{(1/2)(1+\phi)e^2}q_1^2+\frac{1}{(1/2)(1+\phi)e^2}q_2^2=1 \tag{16.2}$$

Symmetric equilibrium solutions imply

$$A-2(\alpha+\beta)q-\frac{2}{(1/2)(1+\phi)e}q=0 \tag{17.1}$$

and;

$$\frac{2}{(1/2)(1+\phi)e^2}q^2=1 \tag{17.2}$$

The only economically admissible solution of (17.2) has;

$$q=\frac{e}{2}\sqrt{1+\phi} \tag{18}$$

Substituting (18) into (17.1), we have;

$$A-(\alpha+\beta)e\sqrt{1+\phi}-\frac{2}{\sqrt{1+\phi}}=0 \tag{19}$$

which yields the following innovation level, decided by the "professional manager";

$$e^I = \frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A - \frac{2}{\sqrt{1+\phi}}\right] \tag{20}$$

The corresponding aggregate production level is;

$$q^I = \frac{1}{(\alpha+\beta)}\left[A - \frac{2}{\sqrt{1+\phi}}\right] \tag{21}$$

Appendix 3: Derivation of the equilibrium outcome of "Joint Surplus Maximization" regime

The joint surplus function is expressed as the total sum of the monetary profit function and the private benefit one, and is given by;

$$\Pi^{JS} = R^I\left(e, q_i, q_j\right) + w_i\left(q_i\right) + w_j\left(q_j\right) \tag{22}$$

which is equivalent to the following equation;

$$R_i\left(e, q_i\right) + R_j\left(e, q_j\right) + w\cdot\left(q_i + q_j\right) = \sum\left(p_i + w\right)q_i - \sum\frac{1}{(1/2)(1+\phi)e}q_i^2 - e \tag{23}$$

The first order conditions of the joint surplus maximization are given by;

$$\frac{\partial\Pi^{JS}\left(e, q_i, q_j\right)}{\partial q_i} = 0,\ \ i, j = 1, 2 \tag{24.1}$$

and

$$\frac{\partial\Pi^{JS}\left(e, q_i, q_j\right)}{\partial e} = 0 \tag{24.2}$$

yielding the following set of equations;

$$\sum\partial\left(A + w - \alpha q_i - \beta q_j\right)q_i \big/ \partial q_i - \frac{2}{(1/2)(1+\phi)e}q_i = 0;\ i, j = 1, 2 \tag{25.1}$$

and;

$$\frac{1}{(1/2)(1+\phi)e^2}q_1^2 + \frac{1}{(1/2)(1+\phi)e^2}q_2^2 = 1 \tag{25.2}$$

These equations reflect the "internalization of externalities" by $q_i, i = 1, 2$ and e.
Hence, we have;

$$\partial\left(A + w - \alpha q_i - \beta q_j\right)q_i \big/ \partial q_i + \partial\left(A + w - \alpha q_j - \beta q_i\right)q_j \big/ \partial q_i - \frac{2}{(1/2)(1+\phi)e}q_i = 0; i, j = 1, 2 \tag{26.1}$$

and;

$$\frac{1}{(1/2)(1+\phi)e^2}q_1^2 + \frac{1}{(1/2)(1+\phi)e^2}q_2^2 = 1 \tag{26.2}$$

Symmetric equilibrium solutions imply

$$A+w-2(\alpha+\beta)q-\frac{2}{(1/2)(1+\phi)e}q=0 \qquad (27.1)$$

and;

$$\frac{2}{(1/2)(1+\phi)e^2}q^2=1 \qquad (27.2)$$

The only economically admissible solution of (30.2) has;

$$q=\frac{e}{2}\sqrt{1+\phi} \qquad (28)$$

Substituting (28) into (27.1), we have

$$A+w-(\alpha+\beta)e\sqrt{1+\phi}-\frac{2}{\sqrt{1+\phi}}=0 \qquad (29)$$

which yields the following innovation level, decided by the "professional manager";

$$e^{JS}=\frac{1}{(\alpha+\beta)\sqrt{1+\phi}}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right] \qquad (30)$$

The corresponding aggregate production level is;

$$q^{JS}=\frac{1}{(\alpha+\beta)}\left[A+w-\frac{2}{\sqrt{1+\phi}}\right] \qquad (31)$$