DISCUSSION PAPER No. 03-E-002

The Price of Debt in a Creditor Coordination Game with Large and Small Creditors

Fumiko Takeda Koichi Takeda

August 2003

The Price of Debt in a Creditor Coordination Game with Large and Small Creditors*

FUMIKO TAKEDA† KOICHI TAKEDA‡

August 2003

Abstract

Large creditors can exert a disproportionate amount of influence in determining the likelihood of debt defaults due to coordination failure. Even if fundamentals are sound, apprehension of premature foreclosure by a single large creditor may trigger preemptive actions by other creditors, and the consequent liquidation of the distressed debtor's assets can result in self-fulfilling debt defaults. To examine the influence of large creditors on the price of defaultable debts, we offer a model in which a large creditor and small creditors independently decide, based on private signals of fundamentals, whether to foreclose on a loan. In the absence of common knowledge of fundamentals, the incidence of failure is uniquely determined. Numerical calculations on the unique equilibrium show that coordination failure among creditors will raise the yield on the debt. However, increase in the large creditor's size and the relative precision of information available to the large creditor may either increase or decrease the yield.

JEL-Classification: G33, G12, D82

Keywords: Large creditors, Creditor coordination, Debt pricing, Incomplete information, Global games

*We would like to thank seminar participants at Institute of Statistical Research for useful comments. Koichi Takeda gratefully acknowledges financial support through Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, and Technology of Japan. The usual disclaimer applies.

†Faculty of Economics and Business Administration, Yokohama City University, 22-2, Seto, Kanazawa-ku, Yokohama 236-0027, Japan. E-mail: ftakeda@yokohama-cu.ac.jp

‡Faculty of Economics, Hosei University, 4342 Aihara-machi, Machida-shi, Tokyo 194-0298, Japan, and Research Center of Socionetwork Strategies, Institute of Economic and Political Studies, Kansai University. E-mail: ktakeda@mt.tama.hosei.ac.jp

1 Introduction

Conventional wisdom holds that large creditors can exert a disproportionate amount of influence in determining the likelihood of debt defaults due to coordination failure. Even if fundamentals are sound, apprehension of premature foreclosure by a single large creditor may induce preemptive actions by other creditors, and the consequent liquidation of the distressed debtor's assets can give rise to self-fulfilling debt defaults.

Our goal in this study is to investigate what influence large creditors have on the value of debts in such a strategic situation. To examine the influence of large creditors in terms of their size and the precision of their information, we employ a theoretical framework building on the global games literature. Global games, first studied by Carlsson and van Damme (1993) , are games with incomplete information whose type space is determined by the players, each of whom observes a private noisy signal of the underlying state.[1] Morris and Shin (1999) applied the global game to the creditor coordination game and related it to the determinants of the value of defaultable debts.[2]

This paper extends the work of Morris and Shin (1999) by introducing a large creditor.[3] Numerical calculations on the unique equilibrium show that coordination failure among creditors will raise the yield on the debt. However, increase in the large creditor's size and the relative precision of information available to the large creditor may either increase or decrease the yield.

The rest of this paper is organized as follows. In Section 2, we set up a creditor coordination game with one large and many small creditors. In Section 3, we derive the unique equilibrium. Section 4 provides numerical calculations in terms of the price of the debt. Concluding remarks are provided in Section 5.

2 The model

Consider an economy in which a project is financed by a continuum of ex-ante identical small creditors and a single large creditor. Each small creditor has an infinitesimal

[1] For global games, see the excellent survey by Morris and Shin (1999) .

[2] Bruche (2003) extends the work of Morris and Shin (1999) and relates it to the continuous-time structural model of bond prices *à la* Merton (1974).

[3] Corsetti *et al.* (2001) , Metz (2002) , and Takeda (2000) use the technique of introducing a large trader into a currency crisis model with a continuum of small traders based on the global game.

portion of the whole stake. There are three periods, $t = 0$, 1, and 2. A project matures at period 2 to yield a return, which is uncertain at period 0. The proportion of loans financed by the large creditor is $\lambda \in (0, 1)$. The combined mass of loans financed by small creditors then amounts to $1 - \lambda$. The face value of repayment is $L > 0$. Each creditor can receive the face value at period 2 if the return is large enough to cover repayment of debt.

At period 1, each creditor has the chance to decide whether to continue lending until the project's maturity, or to stop lending and seize the collateral $K^* \in (0, L)$. If the collateral is liquidated following the project's failure, it has a lower liquidation value $K_* \in [0, K^*)$. The value of the project at maturity depends on two factors: the underlying state θ, which is randomly determined, and a severity of disruption $z > 0$ caused to the project in the event of early liquidation by creditors. Denoting by ℓ the proportion of creditors who stop lending at period 1, if underlying fundamentals θ is larger than $z\ell$, the firm remains in operation. Otherwise, the firm is forced into bankruptcy.

By normalizing the payoffs such that $L = 1$ and $K_* = 0$, the payoffs to a creditor are given by the following matrix, where $\kappa \equiv (K^* - K_*)/(L - K_*)$:

	Project succeeds (if $z\ell < \theta$)	Project fails (if $z\ell \geq \theta$)
Continue lending	1	0
Stop lending	κ	κ

For simplicity, we assume that if continued lending yields the same expected payoff as stopped lending, then a creditor prefers to stop.

The creditors do not observe the realization of θ until period 2, but they receive private signals regarding it. The large creditor receives the signal with the noise η,

$$x_L = \theta + \eta, \tag{1}$$

where η is normally distributed with mean 0 and variance $1/\gamma$. Similarly, a typical small creditor i receives the signal with the noise ε_i,

$$x_i = \theta + \varepsilon_i, \tag{2}$$

where ε_i is normally distributed with mean 0 and variance $1/\beta$. ε_i is i.i.d. across creditors and each is independent of η and θ. The distributions of η and ε_i are assumed to be common knowledge among creditors.

The value of θ is normally distributed with mean y and variance α.[4] Although θ is not common knowledge, both y and α are common knowledge and exogenously given. Upon receiving its respective signal, the representative creditor i can guess the value of θ and the distribution of signals received by the other creditors, as well as their estimates of θ. Likewise, all other creditors form their beliefs while relying only on their own information. This assumption of incomplete information is the key to deriving the unique equilibrium in the global games literature.

The timing of the events is as follows:

Period 0. The firm invests its debt-financed fund into the project.

Nature chooses θ.

Period 1. Signals are observed.

Creditors decide whether to continue lending or to stop lending.

Creditors who choose to stop lending receive κ.

Period 2. Aggregate outcomes are realized.

Creditors who choose to continue lending receive 1 or 0.

3 Equilibrium

Before solving the game described above, we shall briefly discuss the coordination problem when there are complete information regarding θ. If the creditors know the value of θ perfectly before deciding whether to continue lending, their optimal strategies can be described as follows. If $\theta > z$, then to continue lending is optimal, regardless of what the other creditors do, since the project will succeed even if all the other creditors stop lending. In contrast, if $\theta \leq 0$, then to stop lending is optimal regardless of what

[4]This spcification follows the work of Morris and Shin (1999) and that of Metz (2002) . In contrast, Corsetti *et al.* (2001) , Takeda (2000) , and Takeda (2003) assume that θ is uniformly distributed.

the other creditors do, because the project will fail, even if all the other creditors continue lending.

When $\theta \in (0, z]$, there is a coordination problem among the creditors. Each creditor assumes that if all the other creditors continue lending, then the payoff for continued lending is 1, so that continued lending yields more than the early liquidation value κ. However, It assumes that if they stop lending, the payoff is $0 < \kappa$, so that early liquidation is optimal.[5]

We now show that there is a unique, dominance-solvable equilibrium in which both types of creditors follow their respective switching strategies around the critical signals. When small creditor i receives the realization of the signal x_i, its posterior distribution of θ is normal with mean

$$\xi_i \equiv \frac{\alpha y + \beta x_i}{\alpha + \beta}, \tag{3}$$

and variance $1/(\alpha + \beta)$. When small creditors use a switching strategy, they have a threshold level ξ^* for their switching strategies, and continue lending if and only if the private signal x is greater than

$$x^*(\xi^*, y) \equiv \frac{\alpha + \beta}{\beta}\xi^* - \frac{\alpha}{\beta}y.$$

To derive the unique equilibrium, we first consider the critical value of the fundamentals $\overline{\theta}$ above which lending by small creditors alone is sufficient for the project to succeed. $\overline{\theta}$ is given by

$$\overline{\theta} = z\left(1 - (1-\lambda)\Phi\left(\sqrt{\beta}(\overline{\theta} - x^*)\right)\right). \tag{4}$$

Then we consider the critical value of the fundamentals $\underline{\theta}$ above which the project succeeds if and only if both creditors continue lending:

$$\underline{\theta} = z\left(1 - \lambda - (1-\lambda)\Phi\left(\sqrt{\beta}(\underline{\theta} - x^*)\right)\right). \tag{5}$$

[5] This type of coordination problem among creditors is similar to the bank run problem suggested by Diamond and Dybvig (1983) , and entails multiple equilibria in the complete information game where θ is common knowledge.

Since both $\overline{\theta}$ and $\underline{\theta}$ are functions of the switching point x^*, which depends on the large creditor's switching point x_L^*, we need to solve these two switching points simultaneously from the respective optimization problems of the creditors. First, we consider the large creditor's problem. Given $\underline{\theta}$, its optimal strategy is to continue lending if and only if its expected payoff for continued lending conditional on x_L, which is given by $\Phi\left(\sqrt{\alpha+\gamma}\,(\xi_L - \underline{\theta})\right)$, exceeds its payoff for stopped lending, which equals κ. Hence, the switching point $x_L^*(\xi_L^*, y)$ is defined by:

$$\Phi\left(\sqrt{\alpha+\gamma}\,(\xi_L^* - \underline{\theta})\right) = \kappa. \tag{6}$$

Next, we consider a small creditor's problem. If $\theta > \overline{\theta}$, continued lending can be successful regardless of the large creditor's actions. If $\underline{\theta} < \theta \leq \overline{\theta}$, the project succeeds if and only if the large creditor continues lending. If $\theta \leq \underline{\theta}$, the project fails even if the large creditor continues lending. Since a small creditor's optimal strategy is to continue lending if and only if its expected payoff for continued lending conditional on signal x exceeding its payoff for stopped lending, the switching point $x^*(\xi^*, y)$ is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi\left(\sqrt{\alpha+\beta}(\theta - \xi^*)\right)\Phi\left(\sqrt{\gamma}\,(\theta - x_L^*)\right)d\theta + \int_{\overline{\theta}}^{\infty} \phi\left(\sqrt{\alpha+\beta}(\theta - \xi^*)\right)d\theta = \kappa. \tag{7}$$

Proposition 1 *There is a unique, dominance-solvable equilibrium in which the large creditor uses the switching strategy around x_L^* and the small creditors use the switching strategy around x^*.*

The proof of Proposition 1 is given in the Appendix. The unique equilibrium is determined by the four key equations (4), (5), (6), and (7).[6] These equations jointly determine the critical states $\overline{\theta}$ and $\underline{\theta}$ and the switching points x^* and x_L^*.

4 The price of debt

Having established the uniqueness of the equilibrium, we now turn to a set of questions regarding whether the change in the size and the precision of information available to

[6] In contrast to the model with only small creditors discussed in Morris and Shin (1999) , the model with both small and large creditors cannot be solved explicitly for the equilibrium.

the large creditor may increase the value of debts. To address these questions, we provide an argument about the pricing of defaultable debts. Following the work of Morris and Shin (1999) , we consider the value of a long-term unsecured loan with face value 1 to the project. The creditor receives 1 whenever the project succeeds, but receives zero when it fails. Given ex ante mean of fundamentals y, the ex-ante price of such a loan is

$$W(y) = \int_{\underline{\theta}}^{\bar{\theta}} \phi\left(\sqrt{\alpha}\,(\theta - y)\right) \Phi\left(\sqrt{\gamma}\,(\theta - x_L^*)\right) d\theta + \int_{\bar{\theta}}^{\infty} \phi\left(\sqrt{\alpha}\,(\theta - y)\right) d\theta.$$

For simplicity, we define the yield on the unsecured loan as

$$\text{Yield} = \frac{\text{Par} - \text{Price}}{\text{Price}}.$$

Based on the equations above, we can examine how the (1) size of the large creditor λ, (2) precision of the large creditor's signal relative to the small creditors' signals $r \equiv \gamma/\beta$, (3) accuracy of prior information α, and (4) ex-ante mean y affect the yield. We use the following set of variables as a benchmark case for numerical calculations:

$$\alpha = 2,\ \beta = 5,\ \gamma = 5,\ z = 1,\ \kappa = 0.5,\ \lambda = 0.5, \text{ and } y = 2.$$

We first consider the effect on the yield of a shift in the ex-ante mean y. Figure 1 shows that the yield is falling at a rate more than proportional to the increase in y. Comparing the yield from the true model to that from the naive model (no coordination risk), the yield difference becomes large as y falls, indicating an increase in the size of inefficient liquidation due to coordination failure. These results are consistent with those in Morris and Shin (1999) .

We next analyze the impact of the large creditor on the yield. Focusing on the limiting case where both types of creditors have very precise information, Takeda (2003) found that a decrease in the size of the large creditor and an increase in the large creditor's information accuracy reduce the probability of project failure. In contrast, our numerical examples in a more general case indicate that these changes may either reduce or raise the probability of project failure.

To show the effect of size, Figure 1 indicates that the yield from the true model without a large creditor is higher than that with a large creditor when y is low, whereas the former is lower than the latter when y is high. The effect of the size of the large creditor is also affected by the precision of prior information α. Figure 2 shows that the larger the size of the large creditor, the higher the yield. In contrast, Figure 3, with smaller α, shows that the larger the size of the large creditor, the lower the yield.

Finally, increase in the relative precision of information of the large creditor r may either increase or decrease the yield. In Figure 2, the more precise information the large creditor receives, the higher the yield. In contrast, in Figure 3, the more precise information the large creditor receives, the lower the yield.

5 Concluding remarks

We have studied a model of a coordination game played by one large and many small creditors. In the absence of common knowledge of fundamentals, the incidence of failure is uniquely determined. Numerical calculations on the unique equilibrium show that coordination failure among creditors will raise the yield on the debt, and both the large creditor's size and the relative precision of information availabe to the large creditor are important determinants of the yield. However, increase in the large creditor's size and the relative precision of information availabe to the large creditor may either increase or decrease the yield. The results of our study shed light on the importance of the proper evaluation of the influence exerted by large creditors on the value of debts, not only through size channels, but also through information channels.

Appendix

In this appendix, we give a proof of proposition 1. Let us change variables in the integrals as follows:

$$s \equiv \sqrt{\alpha+\beta}\,(\theta-\xi^*)\,,\ \underline{\delta} \equiv \sqrt{\alpha+\beta}\,(\underline{\theta}-\xi^*)\,,\ \text{and}\ \overline{\delta} \equiv \sqrt{\alpha+\beta}\,(\overline{\theta}-\xi^*)\,.$$

Then the conditional expected payoff to continue lending, given signal x^*, is

$$\int_{\underline{\delta}}^{\overline{\delta}} \phi(s)\,\Phi(A)\,ds + \int_{\overline{\delta}}^{\infty} \phi(s)\,ds,$$

where

$$A = \sqrt{\frac{\gamma}{\alpha+\beta}}\left(s - \frac{\alpha+\gamma}{\gamma}\underline{\delta}\right) - \frac{\alpha}{\sqrt{\gamma}}\xi^* + \sqrt{\frac{\alpha+\gamma}{\gamma}}\Phi^{-1}(\kappa) + \frac{\alpha}{\sqrt{\gamma}}y.$$

Hence, the equation (7) is rewritten as

$$\int_{\underline{\delta}}^{\overline{\delta}} \phi(s)\,\Phi(A)\,ds + \int_{\overline{\delta}}^{\infty} \phi(s)\,ds - \kappa = 0. \tag{8}$$

Note, however, that both $\underline{\delta}$ and $\overline{\delta}$ are strictly decreasing in ξ^*, since

$$\frac{d\underline{\delta}}{d\xi^*} = -\frac{1}{z((1-\lambda)\phi(\sqrt{\beta}\underline{\delta}) + 1/\sqrt{\beta}} < 0, \text{ and}$$
$$\frac{d\overline{\delta}}{d\xi^*} = -\frac{1}{z(1-\lambda))\phi(\sqrt{\beta}\overline{\delta}) + 1/\sqrt{\beta}} < 0.$$

Given that the left-hand side of (8) is strictly decreasing in both $\underline{\delta}$ and $\overline{\delta}$, it is strictly increasing in ξ^*. The left-hand side of (8) is negative for sufficiently small ξ^*, and positive for sufficiently large ξ^*. Given that the left-hand side of (8) is continuous in ξ^* there is a unique solution to (8). Once ξ^* is established, the large creditor's threshold ξ_L^* can be determined from (6).

Thus far, we have shown that there is a unique equilibrium within switching strategies. As shown by Takeda (2003) , by the iterative elimination of strictly dominated strategies, the switching strategy identified above can be shown to be the only equilibrium strategy.

References

Bruche, M. (2003) "Corporate Bond Prices and Coordination Failure" *Financial Markets Group Discussion Paper*, 438, London School of Economics.

Carlsson, H., and E. van Damme (1993) "Global Games and Equilibrium Selection" *Econometrica* **61**, 989-1018.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2001) "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders" forthcoming in *Review of Economic Studies.*

Diamond, D., and P. Dybvig (1983) "Bank Runs, Depositor Insurance and Liquidity" *Journal of Political Economy* **91**, 401-419.

Merton, R. C. (1974) "On the Pricing of Corporate Debt: The Risk Structure of Interest Rate" *Journal of Finance* **29**, 449-470.

Metz, C. E. (2002) "Currency Crises: The Role of Large Traders" Volkswirtschaftliche Diskussionsbeiträge, 28/02, University of Kassel.

Morris, S., and H. S. Shin (1999) "Coordination Risk and the Price of Debt" forthcoming in *European Economic Review.*

Morris, S., and H. S. Shin (2002) "Global Games: Theory and Applications" in *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, edited by M. Dewatripont, L. P. Hansen and S. J. Turnovsky, Cambridge University Press.

Takeda, F. (2000) "A Twin Crisis Model with Incomplete Information" forthcoming in *Journal of the Japanese and International Economies.*

Takeda, K. (2003) "The Role of Large Creditors in Creditor Coordination Games" ICES Discussion Paper, 03-E-001, Institute of Comparative Economic Studies, Hosei University.

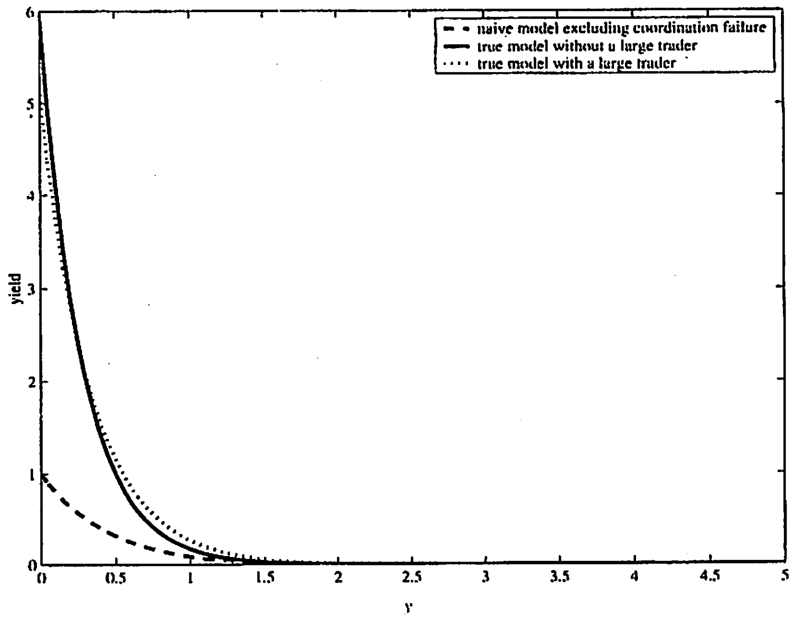

Figure 1 Effect of y on the yield

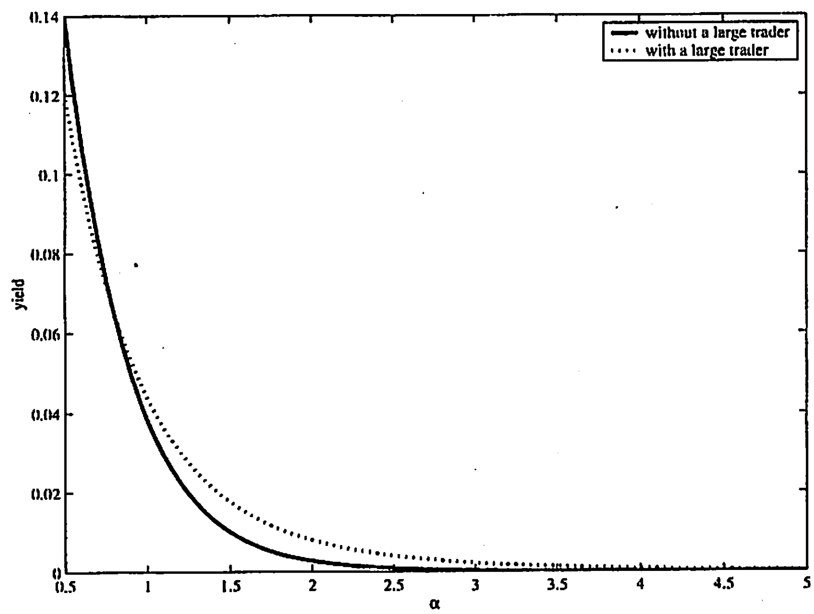

Figure 2 Effect of λ on the yield ($\alpha = 2$)

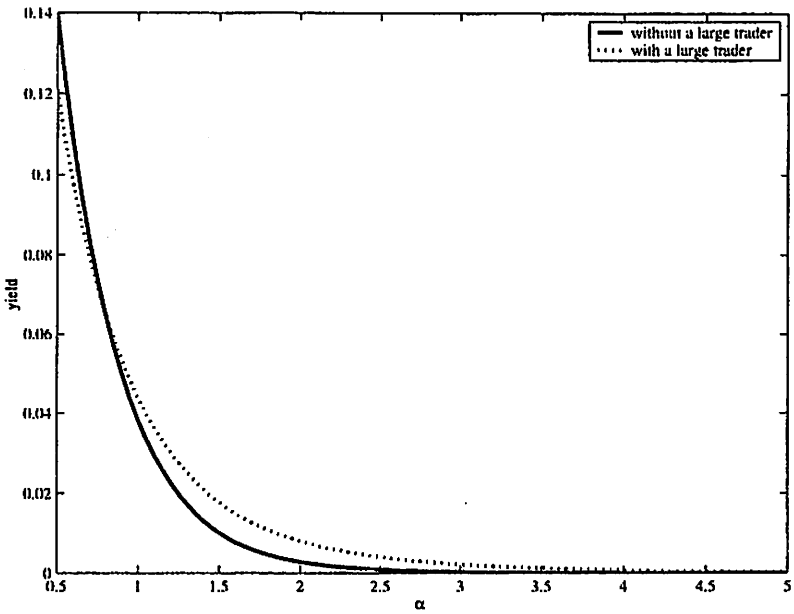

Figure 3 Effect of λ on the yield ($\alpha = 0.1$)