What Prices Should be Targeted by a Central Bank?

-A Case in VAT increase-[1]

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Abstract

This paper investigates what prices, the consumer price or the producer one, should be targeted by a central bank when the government increases the tax rate on the consumption goods, i.e. Value added tax (VAT). We compare with two policies using New-Keynesian DSGE model with the producer price stickiness. We see that the producer price, which means the price without tax, targeting better off when the producer price is flexible and the VAT increase shock is less persistency. On the other hand, consumer price target which includes the tax may be better off when the producer price is sticky or the VAT increase shock is quite persistency.

Key words: Value Added Tax (VAT), New Keynesian DSGE model, Producer piece stickiness.
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1. Introduction

Taxing on the consumption goods, namely the consumption tax or value added tax (VAT), accepts a lot of countries. Recently, several countries, which face the sovereign problem, raise its tax rate. For example, Greece raised the VAT rate from 18% to 23% in 2015, and Italy also raised it from 21% to 23% in 2012. Increasing tax rate on consumption causes decline in the consumption and output and increase in the consumer price. In this case, the monetary authorities face the difficult situation of the trade-off between stabilizing the output and the inflation rate. In fact, the monetary policy treatments under the VAT increase are various. Figure 1 and 2 show that the Bank of Japan (BOJ) and Bank of England (BOE) take the different policy stance to each case in VAT (or consumption tax rate) increase. For example, the BOJ increased the official bank rate from 2.5% to 3% in April 1989 that the consumption tax induces and set its rate as 3%. On the other hand, the BOJ cut lower the official bank rate from 12.375% to 11.875% in January 1991, the VAT increases from 15% to 17.5%.

This paper investigates which consumer price index (CPI) or producer price index (PPI) should be targeted by a central bank when the government increases the tax rate on the consumption goods. Concretely, this paper compares with CPI which includes the component of value added tax (VAT) and PPI which does not include it. To compare with two policies, we use the New Keynesian DSGE model with consumption tax. Introducing the consumption tax, we can compare with the consumer price, which includes the tax and the producer price; i.e. the price without tax. We obtain two remarkable results. First, the economy under both the consumer and the producer price targeting follow the resemble response after tax increase. On the other hand, these are qualitative and quantitative difference of responses of output and inflation rate. Second, comparing with two monetary policies with respect to welfare, order between them is not monotonic. Concretely, the producer price targeting is better off, if the producer price is flexible and the persistency of VAT increase shock is low.

There are several related literatures about the effect of consumption tax in DSGE model and investigation about the inflation rate of the Taylor rule. As for the consumption tax increase in DSGE model, we refer to Forni et al. (2009) and Iwata (2011) which use the Consumer Price Index (CPI) based Taylor rule. As for the consideration about the inflation in Taylor rule, Aoki

(2001) considers about multi sector model which compose flexible price sector and sticky price one, and show that the optimal monetary policy is to target sticky price inflation. Benigno (2004), Bernanke and Woodford (2000), Corsetti, Dedola and Leduc (2007) and Okano (2007) analyze the optimal inflation target in New Keynesian open-economy macroeconomics[2].

The paper is organized as follows. In Section 2, we describe the DSGE model with consumption tax. In Section 3, we analysis the model under unanticipated tax increase and Section 4 expands the case in anticipated tax increase. Section 5 concludes the paper.

2. The Model

In this model, we construct the DSGE model with ad-valorem taxes. This paper sets the sticky price DSGE model a la Calvo (1983) and we do not include physical capital. Similar to prototype DSGE model, the intermediate firm faces a monopolistic competition and can change the producer price with probability $1-\rho$, while remain its price with probability ρ. The government distributes the lump-sum transfer finance by both unit and ad-valorem tax. That is, increase in unit or ad-valorem tax occur only the substitution effect. For simplicity, we assume that the inflation rate at the steady state sets zero. We set the lifetime utility is a separable function as follow:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda} \right],$$

$$c_t = \left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where θ is the demand elasticity to price, and $\dfrac{\theta}{\theta-1}$ is the markup.

We consider a prototype DSGE model with consumption tax and transfer from the government to the households[3].

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[2] Bernanke and Woodford (2000) shows the producer price inflation target is better under the assumption that purchasing power parity applies. While Okano (2007) shows that the consumer price inflation target is better under the assumption that the pricing to market applies.

[3] We explain the detail of the model in Appendix.

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - \hat{\pi}_{C,t+1}\right), \tag{1}$$

$$\hat{\pi}_{P,t} = \frac{(1-\rho)(1-\beta\rho)}{\rho}(\sigma+\lambda)\hat{y}_t + \frac{(1-\rho)(2-\beta\rho)}{\rho}\frac{\tau}{1+\tau}\hat{\tau}_t + \beta\hat{\pi}_{P,t+1}. \tag{2}$$

$$\hat{\pi}_{c,t} = \hat{\pi}_{p,t} + \frac{\tau}{1+\tau}\left(\hat{\tau}_t - \hat{\tau}_{t-1}\right), \tag{3}$$

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_\pi \hat{\pi}_{j,t}, \quad (j = C, P) \tag{4}$$

$$\hat{\tau}_t = \omega\hat{\tau}_{t-1} + \varepsilon_t, \tag{5}$$

where $\hat{y}_t$ is the output gap, $\hat{\pi}_{C,t}$ is the inflation rate of the consumer price (i.e. tax including price) deviated from the steady state, $\hat{\pi}_{P,t}$ is the inflation rate of the consumer price (i.e. tax including price) deviated from the steady state, σ is the relative risk aversion, λ is the inverse of the Frisch labor supply, β is the discount factor, ρ is the probability of remaining price, $\hat{\tau}$ is the consumption tax rate, $\hat{i}$ is the policy rate, ϕ_y and ϕ_π is the parameter of the Taylor rule, and ν_t is an anticipated consumption tax shock.

Eq. (1) represents the New-Keynesian IS (NKIS) curve. Eq. (2) represents the New-Keynesian Phillips curve (NKPC). Eq. (3) defines the relationship between the consumer price and the producer price. Eq. (4) represents the Taylor rule, which considers about the inflation target with respect to the price level; the consumer price or the producer one. Eq. (5) represents the law of motion of the VAT rate which assumes AR (1) process, ω is the persistency parameter and ε is the tax shock.

3. Analysis of the model

We discuss about the comparison with the two price targets when the unanticipated VAT increase causes. That is, consumers and firms know the tax rate increase in period 1 and cannot in previous period.

3.1. The Producer Price target

First, we consider about the case in the producer price target. That is, we define the Taylor rule as follow:

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_\pi \hat{\pi}_{P,t}.$$

Substituting above the Taylor rule to Eq. (1), we can obtain the following equation:

$$\hat{y}_t = \hat{y}_{t+1} - \frac{1}{\sigma}\left(\phi_y \hat{y}_t + \phi_\pi \hat{\pi}_{P,t} - \hat{\pi}_{P,t+1} + \frac{\tau(1-\omega)}{1+\tau}\hat{\tau}_t \right). \tag{6}$$

Using guess and verify method, we obtain the solution of the model at period t=2 and later as follows:

$$\hat{y}_t = \frac{\tau[(1-\omega)(1-\beta\omega) - \alpha_2(\phi_\pi + \omega)]}{1+\tau}\Delta^{-1}\hat{\tau}_t \equiv \psi_y \hat{\tau}_t, \tag{7}$$

$$\hat{\pi}_{P,t} = (1-\beta\omega)^{-1}\left[\alpha_1(\sigma+\lambda)\psi_y + \frac{\tau\alpha_2}{1+\tau} \right]\hat{\tau}_t \equiv \psi_\pi \hat{\tau}_t, \tag{8}$$

$$\hat{\pi}_{c,t} = \begin{cases} \left[\psi_\pi - \dfrac{\tau(1-\omega)}{(1+\tau)\omega} \right]\hat{\tau}_t & if \quad t \geqq 2 \\ \left(\psi_\pi + \dfrac{\tau}{1+\tau} \right)\hat{\tau}_t & if \quad t = 1 \end{cases}, \tag{9}$$

where

$$\Delta \equiv (1-\beta\omega)[(1-\omega)\sigma + \phi_y] + (\sigma+\lambda)\alpha_1(\phi_\pi + \omega) > 0, \alpha_1 \equiv \frac{(1-\rho)(1-\beta\rho)}{\rho}, \alpha_2 \equiv \frac{(1-\rho)(2-\beta\rho)}{\rho}.$$

We see $\psi_\pi > 0$, while ψ_y is positive (negative) if $\alpha_2 < (>) \frac{(1-\omega)(1-\beta\omega)}{\phi_\pi + \omega}$ [4]. In this case, we obtain the same policy function at the period t=1, since the NKIS curve at period 1 is same as that at period t≧2. There is remarkable feature about the response of PPI. Eq .(8) shows that the PPI inflation rate increases if $\alpha_1(\sigma+\lambda)\psi_y + \frac{\tau\alpha_2}{1+\tau} > 0$. Although it is different from the result of partial analysis, it may be consistent with the actual data shown in Figure 3.

As for the nominal interest rate $\hat{i}_t$, we can obtain the following equation:

$$\hat{i}_t = [\phi_y \psi_y + \phi_\pi \psi_\pi]\hat{\tau}_t = \left\{ \left[\phi_y + \frac{\alpha_1(\sigma+\lambda)\phi_\pi}{1-\beta\omega} \right]\psi_y + \frac{\phi_\pi \alpha_2 \tau}{(1-\beta\omega)(1+\tau)} \right\}\hat{\tau}_t.$$

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[4] In other word, ψ_y is positive (negative) if $\phi_\pi < (>) \frac{\rho(1-\omega)(1-\beta\varpi)}{(1-\rho)(2-\beta\rho)} - \omega$.

3.2. Consumer Price Target

We consider about the case in the producer price target. That is, we define the Taylor rule as follow:

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_\pi \hat{\pi}_{C,t}.$$

Substituting above equation to Eq. (1), we can obtain the following equation:

$$\begin{aligned} \hat{y}_t &= E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(\phi_y \hat{y}_t + \phi_\pi\left(\hat{\pi}_{P,t} + \frac{\tau}{1+\tau}(\hat{\tau}_t - \hat{\tau}_{t-1})\right) - E_t \hat{\pi}_{P,t+1} - \frac{\tau}{1+\tau}(\hat{\tau}_{t+1} - \hat{\tau}_t)\right). \\ \Leftrightarrow (\sigma+\lambda)\hat{y}_t &= \sigma E_t \hat{y}_{t+1} - \phi_\pi \hat{\pi}_{P,t} - E_t \hat{\pi}_{P,t+1} - \frac{\tau(1-\omega)(\phi_\pi - \omega)}{(1+\tau)\omega}\hat{\tau}_t. \end{aligned} \tag{10}$$

Similar to the previous subsection, we use the guess and verify method to solve the policy function of output and two inflation rates at period t≧2 as follows:

$$\begin{aligned} \hat{y}_t &= -\frac{\tau\left[(1-\omega)(1-\beta\omega)(\phi_\pi - \omega)\omega^{-1} + \alpha_2(\phi_\pi + \omega)\right]}{1+\tau}\Delta^{-1}\hat{\tau}_t \\ &= \left[\psi_y - \frac{\tau(1-\omega)(1-\beta\omega)\phi_\pi}{(1+\tau)\omega\Delta}\right]\hat{\tau}_t \equiv \chi_y \hat{\tau}_t, \end{aligned} \tag{11}$$

$$\hat{\pi}_{P,t} = (1-\beta\omega)^{-1}\left[\alpha_1(\sigma+\lambda)\chi_y + \frac{\tau\alpha_2}{1+\tau}\right]\hat{\tau}_t \equiv \chi_\pi \hat{\tau}_t, \tag{12}$$

$$\hat{\pi}_{c,t} = \left[\psi_\pi - \frac{\tau(1-\omega)}{(1+\tau)\omega}\right]\hat{\tau}_t. \tag{13}$$

On the policy functions at period 1 are induced as follow:

$$\begin{aligned} \hat{y}_1 &= \left[\sigma + \phi_y + \phi_\pi \alpha_1(\sigma+\lambda)\right]^{-1}\left[\chi_y - (1+\beta\phi_\pi)\chi_\pi + \frac{(1-\alpha_2)\tau}{1+\tau}\right]\hat{\tau}_t \\ &\equiv \chi_{y1}\hat{\tau}_t, \end{aligned} \tag{14}$$

$$\hat{\pi}_{P,1} = \left(\alpha_1(\sigma+\lambda)\chi_{y1} + \beta\chi_\pi + \frac{\alpha_2\tau}{1+\tau}\right)\hat{\tau}_t \equiv \chi_{\pi 1}\hat{\tau}_t, \tag{15}$$

$$\hat{\pi}_{C,1} = \left(\alpha_1(\sigma+\lambda)\chi_{y1} + \beta\chi_\pi + \frac{(1+\alpha_2)\tau}{1+\tau}\right)\hat{\tau}_t. \tag{16}$$

Eq. (11), (12), (13), (14), (15) and (16) explain that the dynamic processes of output and inflation rate follow the policy functions in Eq. (11), (12) and (13), except for the case in period 1 shown in Eq. (14), (15) and (16).

3.3. Welfare Comparison

Similar to Rotemberg and Woodford (1997), a second order approximation to the welfare function (equal to the household's utility function) around the steady state:

$$U_0 \cong -\frac{U_C y}{2} E_0 \sum_{T=0}^{\infty} \beta^T \left[\alpha_1^{-1} \hat{\pi}_{C,T}^2 + (\sigma+\lambda)\hat{y}_T^2\right], \tag{17}$$

where $U_C y = y^{1-\sigma} = \left(\frac{\theta-1}{(1+\tau)\theta}\right)^{1/(\sigma+\lambda)}$.

We substitute Eq. (11) ,(12), (13), (14), (15) and (16) to Eq. (17) to compare with the two policy targets under VAT increase.

$$U_{0,PT} = -\frac{U_C y\beta}{2}\left[\begin{array}{l}\frac{1}{\alpha_1}\left(\psi_\pi + \frac{\tau}{1+\tau}\right)^2 + \frac{\beta\omega^2}{\alpha_1(1-\beta\omega^2)}\left(\psi_\pi - \frac{\tau(1-\omega)}{(1+\tau)\omega}\right)^2 \\ + \frac{(\sigma+\lambda)\omega^2\psi_y^2}{1-\beta\omega^2}\end{array}\right]\hat{\tau}_t^2, \tag{18}$$

$$U_{0,CT} = -\frac{U_C y\beta}{2}\left[\begin{array}{l}\frac{1}{\alpha_1}\left(\chi_{\pi,1} + \frac{\tau}{1+\tau}\right)^2 + \frac{\beta\omega^2}{\alpha_1(1-\beta\omega^2)}\left(\psi_\pi - \frac{\tau(1-\omega)}{(1+\tau)\omega}\right)^2 \\ (\sigma+\lambda)\chi_{y,1}^2 + \frac{(\sigma+\lambda)\beta\omega^2\chi_y^2}{1-\beta\omega^2}\end{array}\right]\hat{\tau}_t^2, \tag{19}$$

where $U_{0,PT}$ is the welfare under producer price target, and $U_{0,CT}$ is that under consumer price target.

First and second terms of parenthesis of the right hand side represent the inflation volatility in each price target, and third (and fourth in Eq. (19)) term are the output volatility.

3.4. Numerical Example

In this subsection, we make the numerical example to investigate which policy is better off using somewhat valid parameter values. We show the benchmark parameter in Table 1[5]. Figure 4 and 5 show the impulse response

[5] We refer to some calibrated or estimated parameters (e.g. Smets and Wouters (2003, 2007), Sugo and Ueda (2007), Iwata (2011) etc.) when we decide to these parameters

of output and each inflation rate to unanticipated VAT increase shock at period 1. Shown in Figure 4 and 5, the responses of output are qualitatively different and that of PPI target is less volatile than that of CPI target, while there is little difference among CPI and PPI targets. Figure 5 shows the welfare comparison with two monetary policy stances and PPI target is dominated to CPI target in benchmark parameters. In Figure 5, we see two remarkable points. First, the producer price target is usually better off. This result may be similar to the result of Aoki (2001). Aoki (2001) shows the optimal monetary policy in multi-sector model focuses on the sector that faces on the price stickiness[6]. Second, the CPI targeting rule is better off when the producer price is sticky or (and) the persistency of tax shock is high. Figure 6, 7, 8 and 9 show the initial responses of output and CPI inflation which affect the welfare in each ρ and ω. Especially, the higher ρ is, the better the CPI targeting is because the output volatility of CPI targeting is much less than that of PPI targeting. As for the persistency of VAT shock ω, it affects the relative price between CPI and PPI change at period $t \geqq 2$. Shown in Eq. (3), CPI is smaller than PPI[7].

4. Extension to the anticipated VAT increase

In this section, we extend to the case in more realistic case; i.e. the anticipated VAT increase. We represent the anticipated VAT shock υ_t as follow:

$$\hat{\tau}_{t+1} = \omega\hat{\tau}_t + \varepsilon_{t+1} + \upsilon_t .$$

In this case, we take an anticipated VAT shock at period 0; i.e. $\upsilon_0 = 1$. Then , we rewrite the NKIS and NKPC at period 0 as follows[8]:

$$\hat{y}_0 = \hat{y}_1 - \frac{1}{\sigma}\left(\hat{i}_0 - \hat{\pi}_{P,1} - \frac{\tau}{1+\tau}\hat{\tau}_t\right), \tag{20}$$

(Especially, we refer to Iwata (2011) with respect to seminal parameters ω and ρ). Although this case is merely numerical example, the result in this paper is not so far from the realistic.

[6] We need to pay attention to the difference of the definition of the CPI. Aoki (2001) define CPI as composite of flexible and sticky producer goods.

[7] As for σ and λ, there is no quantitative difference from benchmark setting even if we change them as other reasonable parameter values.

[8] As for the unanticipated shock, there is no deviation from the steady state at the period 0; i.e. $\hat{y}_0 = E_0\hat{y}_1 = \hat{\pi}_{P,0} = E_0\hat{\pi}_{P,1} = 0$.

$$\hat{\pi}_{P,0} = (\sigma+\lambda)\alpha_1\hat{y}_0 + \beta\hat{\pi}_{P,1}, \tag{21}$$

and $\hat{\pi}_{C,0} = \hat{\pi}_{P,0}$. We obtain the closed form of $\hat{y}_0, \hat{\pi}_{P,0}$:

$$\hat{y}_0 = \frac{\sigma\hat{y}_1 - \beta\phi_\pi\hat{\pi}_{P,1} + \frac{\tau}{1+\tau}\hat{\tau}_t}{\mathrm{K}}, \tag{22}$$

$$\hat{\pi}_{P,0} = \frac{\beta[\mathrm{K} - \phi_\pi\alpha_1(\sigma+\lambda)]}{\mathrm{K}}\hat{\pi}_{P,1} + \frac{\alpha_1(\sigma+\lambda)}{\mathrm{K}}\left(\sigma\hat{y}_1 + \frac{\tau}{1+\tau}\hat{\tau}_t\right), \tag{23}$$

where $\mathrm{K} \equiv \sigma + \phi_y + \alpha_1\phi(\sigma+\lambda)$, $\hat{y}_1 = \psi_y$ (if PPI rule), or χ_y (CPI rule),

$\hat{\pi}_{P,1} = \psi_\pi$ (if PPI rule), or $\chi_{\pi 1}$ (CPI rule).

Substituting Eq. (14)-(17) to Eq. (12), we obtain the welfare under PPI and CPI target equation as following two equations:

$$U_{0,PT} = -\frac{U_C y}{2}\begin{bmatrix} \frac{\hat{\pi}_{P,0}^2}{\alpha_1} + \frac{\beta}{\alpha_1}\left(\psi_\pi + \frac{\tau}{1+\tau}\right)^2 + \frac{\beta^2\omega^2}{\alpha_1(1-\beta\omega^2)}\left(\psi_\pi - \frac{\tau(1-\omega)}{(1+\tau)\omega}\right)^2 \\ (\sigma+\lambda)\hat{y}_o^2 + \frac{\beta(\sigma+\lambda)\omega^2\psi_y^2}{1-\beta\omega^2} \end{bmatrix}\hat{\tau}_t^2, \tag{24}$$

$$U_{0,CT} = -\frac{U_C y}{2}\begin{bmatrix} \frac{\hat{\pi}_{P,0}^2}{\alpha_1} + \frac{\beta}{\alpha_1}\left(\chi_{\pi,1} + \frac{\tau}{1+\tau}\right)^2 + \frac{\beta^2\omega^2}{\alpha_1(1-\beta\omega^2)}\left(\psi_\pi - \frac{\tau(1-\omega)}{(1+\tau)\omega}\right)^2 \\ +(\sigma+\lambda)\hat{y}_0^2 + \beta(\sigma+\lambda)\chi_{y,1}^2 + \frac{(\sigma+\lambda)\beta^2\omega^2\chi_y^2}{1-\beta\omega^2} \end{bmatrix}\hat{\tau}_t^2, \tag{25}$$

Using Eq. (20), (21), (22) and (23), we obtain the impulse responses of output and inflation rate to anticipated VAT increase shock shown in Figure 11 and 12. Figure 11 and 12 show that the quantitative difference between CPI and PPI targets does not cause well. As a result, the result of the welfare comparison is similar to the case in unanticipated shock[9].

5. Conclusion

This paper investigates which prices, i.e. consumer or producer price should

[9] Actually, welfare comparison with two monetary policy under anticipated VAT shock is almost same in Figure 5.

be targeted by a central bank when the government increases the tax rate on the consumption goods. To compare with two policies, we use the New Keynesian DSGE model with consumption tax. Introducing the consumption tax, the difference among the consumer price and the producer one causes. We obtain two remarkable results. First, the economy under both the consumer and the producer price targeting follow the resemble response after tax increase. On the other hand, these are qualitative and quantitative difference of responses of output and inflation rate. Second, comparing with two monetary policies with respect to welfare, order between them is not monotonic. Concretely, the producer price targeting is better off, if the producer price is flexible or very sticky.

There are several extensions of this paper. For example, we apply the richer DSGE model, such as introducing capital, consumption externality, wage rigidity etc. In addition, we may analyze the optimal consumption tax in DSGE model. These possibilities are to be addressed in future researches.

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Appendix: Derivation of the model

A.1. The Households

Lifetime utility of the representative households is a separable function of his or her consumption index c_t, which consists of each goods $c_{j,t}$ where $j \in (0,1)$ and labor l_t given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda} \right], \tag{A.1}$$

$$c_t = \left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where θ is the demand elasticity to price, and $\frac{\theta}{\theta-1}$ is the price markup.

The nominal flow budget constraint is given by:

$$W_t l_t + (1+i_{t-1}) B_{t-1} + V_t = P_{C,t} c_t + B_t, \tag{A.2}$$

where the consumer price index is $P_{C,t} = (1+\tau_t) P_{P,t}$, $P_{P,t}$ is the producer price index.

First, we reset the utility maximization problem to cost minimization problem as follow:

$$\begin{aligned} &\min_{\{c_{h,t}\}} \int_0^1 p_{C,j,t} c_{j,t} \\ &s.t. \quad c_t - \left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = 0, \end{aligned} \tag{A.3}$$

where $p_{C,j,t}$ is the consumer price of good j which is equal to $(1+\tau_t) p_{P,j,t}$.

We obtain the following equations w.r.t. the consumer price index $P_{C,t}$ and $c_{j,t}$;

$$P_{C,t} = \left[\int_0^1 p_{C,j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{A.4}$$

$$c_{j,t} = \left(\frac{p_{C,j,t}}{P_{C,t}} \right)^{-\theta} c_t . \tag{A.5}$$

Next, using Lagrangean which maximizes (A.1) subjected to (A.2), we obtain the following equations:

$$\mu_t = c_t^{-\sigma}, \tag{A.6}$$

$$\frac{W_t \mu_t}{P_{C,t}} = l_t^{\lambda}, \tag{A.7}$$

$$\beta(1+i_t)\left(\frac{\mu_{t+1}}{P_{C,t+1}} \right) = \frac{\mu_t}{P_{C,t}}, \tag{A.8}$$

Where μ_t is the Lagrange multiplier of (A.2).

A.2. The firms

The production technology of firm j is given by:

$$y_{j,t} = l_{j,t} . \tag{A.9}$$

The firm j minimizes the real term of total cost $\frac{W_t}{P_{C,t}}$. Therefore, we can obtain following condition:

$$\eta_t = \frac{W_t}{P_{C,t}} \equiv mc_t, \tag{A.10}$$

where mc_t is the real marginal cost.

Each firm *j* can reset the producer price $p_{p,j,t}$ with probability $1-\rho$.

Then, the firm j which can reset his price sets $\tilde{p}_{P,j,t}$ to maximize the present value of profit as follow:

$$E_t \sum_{i=0}^{\infty} \rho^i \prod_{k=0}^{i} (1+i_{t+k})^{-1} (p_{P,j,t} - P_{C,t+k} mc_{t+k}) y_{j,t+k}, \tag{A.11}$$

Substituting (A.5) and resource constraint $y_t = l_t = c_t$ to (A.8),

$$
\begin{aligned}
&E_t \sum_{i=0}^{\infty} \rho^i \prod_{k=0}^{i} (1+i_{t+k})^{-1} \left(p_{P,j,t} - P_{C,t+k} mc_{t+k}\right) \left(\frac{p_{C,j,t}}{P_{C,t}} \right)^{-\theta} y_t \\
&= E_t \sum_{i=0}^{\infty} \rho^i \prod_{k=0}^{i} (1+i_{t+k})^{-1} P_{C,t+k} \left(\frac{p_{P,j,t}}{(1+\tau_{t+k}) P_{P,t+k}} - mc_{t+k} \right) \left(\frac{p_{P,j,t}}{P_{P,t+k}} \right)^{-\theta} y_t
\end{aligned} \tag{A.12}
$$

Substituting (A.8) to (A.12),

$$
(1+\tau_t) P_{C,t} y_t E_t \sum_{i=0}^{\infty} \beta^i \rho^i \left(\frac{1}{1+\tau_{t+k}} \left(\frac{\tilde{p}_{P,j,t}}{P_{P,t+k}} \right)^{1-\theta} - mc_{t+k} \left(\frac{\tilde{p}_{P,j,t}}{P_{P,t+k}} \right)^{-\theta} \right), \tag{A.13}
$$

Differentiating (A.13) to $\tilde{p}_{p,j,t}$,

$$
\begin{aligned}
&P_{C,t} y_t \sum_{i=0}^{\infty} \beta^i \rho^i \left((1-\theta) \frac{\tilde{p}^{\theta}_{P,j,t}}{P^{1-\theta}_{P,t+i}} + \theta (1+\tau_{t+k}) mc_{t+k} \frac{\tilde{p}^{\theta-1}_{P,j,t}}{P^{\theta}_{P,t+i}} \right) = 0, \\
&\Leftrightarrow \tilde{p}_{p,j,t} \sum_{i=0}^{\infty} \beta^i \rho^i P^{\theta-1}_{P,t+i} = \frac{\theta}{\theta-1} \sum_{i=0}^{\infty} \beta^i \rho^i (1+\tau_{t+k}) mc_{t+i} P^{\theta}_{P,t+i} \\
&\Leftrightarrow \frac{\tilde{p}_{P,j,t}}{P_t} = \frac{\theta}{(\theta-1)(1+\tau_t)} \frac{\sum_{i=0}^{\infty} \beta^i \rho^i (1+\tau_{t+k}) mc_{t+i} \, P_{P,t+i} / P_{P,t}}{\sum_{i=0}^{\infty} \beta^i \rho^i \, P_{P,t+i} / P_{P,t}}.
\end{aligned} \tag{A.14}
$$

We define $F_t = \sum_{i=0}^{\infty} \beta^i \rho^i mc_{t+i} \, P_{P,t+i} / P_{P,t}, Z_t = \sum_{i=0}^{\infty} \beta^i \rho^i \, P_{P,t+i} / P_{P,t}$, we obtain the following equations:

$$
F_t = (1+\tau_{t+k}) mc_t + \beta\rho \left(\frac{P_{P,t+1}}{P_{P,t}} \right)^{\theta} F_{t+1}, \tag{A.15}
$$

$$
Z_t = 1 + \beta\rho \left(\frac{P_{P,t+1}}{P_{P,t}} \right)^{\theta} Z_{t+1}, \tag{A.16}
$$

$$
\frac{\tilde{p}_{P,j,t}}{P_{P,t}} = \frac{\theta}{\theta-1} \frac{F_t}{Z_t}. \tag{A.17}
$$

Using the probability ρ, we rewrite (A.4) as follow:

$$
\begin{aligned}
&P_{C,t} = \left[\int_0^1 p^{1-\theta}_{C,j,t} dj \right]^{\frac{1}{1-\theta}} \Leftrightarrow P_{P,t} = \left[\int_0^1 p^{1-\theta}_{P,j,t} dj \right]^{\frac{1}{1-\theta}} \\
&\Leftrightarrow P^{1-\theta}_t = (1-\rho) \tilde{p}^{1-\theta}_t + \rho P^{1-\theta}_{t-1}.
\end{aligned} \tag{A.18}
$$

A.3. Monetary Policy

The central government sets a policy interest rate following the Taylor rule:

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_\pi \hat{\pi}_{j,t}, \quad (j = C, P)$$

In this paper, we compare with two price target of consumer and produce prices.

A.4. The Government

The government levies consumption tax and pay back to households as a lump-sum transfer V_t.

$$\tau_t P_{P,t} c_t = V_t.$$

A.5. The equilibrium conditions

Using above equations and each inflation rate $\pi_{C,t+1} = \dfrac{P_{C,t+1}}{P_{C,t}}, \pi_{P,t+1} = \dfrac{P_{P,t+1}}{P_{P,t}}$ and $\tilde{\pi}_{p,j,t+1} = \dfrac{\tilde{p}_{P,j,t+1}}{\tilde{p}_{P,j,,t}}$, we obtain the equilibrium conditions as follow:

$$mc_t = y_t^{\sigma+\lambda}, \tag{A.19}$$

$$\left(\frac{y_{t+1}}{y_t}\right)^\sigma = \beta(1+i_t)\pi_{C,t+1}, \tag{A.20}$$

$$\frac{\tilde{\pi}_{P,j,t}}{\pi_{P,t}} = \frac{\theta(1+\tau_t)}{\theta-1}\frac{F_t}{Z_t}, \tag{A.21}$$

$$F_t = (1+\tau_t)mc_t + \beta\rho\pi_{P,t+1}^\theta F_{t+1}, \tag{A.22}$$

$$Z_t = 1 + \beta\rho\pi_{P,t+1}^\theta Z_{t+1}, \tag{A.23}$$

$$\pi_{P,t}^{1-\theta} - \rho = (1-\rho)\tilde{\pi}_{P,t}^{1-\theta}, \tag{A.24}$$

We obtain the steady state values as follow:

$$\pi_P = \pi_C = \tilde{\pi}_{P,j} = 1,$$

$$i = \frac{1-\beta}{\beta},$$

$$Z = \frac{1}{1-\beta\rho},$$

$$mc = \frac{\theta-1}{\theta},$$

$$F = \frac{(\theta-1)(1+\tau)}{\theta(1-\beta\rho)},$$

$$y = h = c = \left[\frac{\theta-1}{(1+\tau)\theta}\right]^{\frac{1}{\sigma+\lambda}}.$$

A.6. Log-linearized equilibrium

Log-linearized around the steady state in (A.15), (A.16), (A.18), (A.19) and (A.20), we can obtain the following equations:

$$m\hat{c}_t = (\sigma+\lambda)\hat{y}_t, \tag{A.25}$$

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - E_t\hat{\pi}_{C,t+1}\right), \tag{A.26}$$

$$\hat{\tilde{\pi}}_{P,t} - \hat{\pi}_{P,t} = \frac{\tau}{1+\tau}\hat{\tau}_t + \hat{F}_t - \hat{Z}_t, \tag{A.27}$$

$$\hat{\pi}_{P,t} = (1-\rho)\hat{\tilde{\pi}}_{P,t}, \tag{A.28}$$

$$\hat{F}_t = (1-\beta\rho)\left(m\hat{c}_t + \frac{\tau}{1+\tau}\hat{\tau}_t\right) + \beta\rho\left(\theta\hat{\pi}_{P,t} + \hat{F}_{t+1}\right), \tag{A.29}$$

$$\hat{Z}_t = \beta\rho\left[(\theta-1)\hat{\pi}_{P,t+1} + \hat{Z}_{t+1}\right]. \tag{A.30}$$

Substituting (A.29) and (A.30) to (A.27),

$$\hat{\tilde{\pi}}_{P,t} - \hat{\pi}_{P,t} = (1-\beta\rho)m\hat{c}_t + \frac{(2-\beta\rho)\tau}{1+\tau}\hat{\tau}_t + \beta\rho\hat{\tilde{\pi}}_{P,t+1} \tag{A.31}$$

We eliminate $\hat{\tilde{\pi}}_{P,t}$ using (A.28),

$$\hat{\pi}_{P,t} = \frac{(1-\rho)(1-\beta\rho)}{\rho} m\hat{c}_t + \frac{(1-\rho)(2-\beta\rho)}{\rho} \frac{\tau}{1+\tau} \hat{\tau}_t + \beta \hat{\pi}_{P,t+1}. \tag{A.32}$$

$$\hat{\pi}_{c,t} = \hat{\pi}_{p,t} + \frac{\tau}{1+\tau} (\hat{\tau}_t - \hat{\tau}_{t-1}). \tag{A.33}$$

Substituting (A.25) to (A.32),

$$\hat{\pi}_{P,t} = \frac{(1-\rho)(1-\beta\rho)}{\rho} (\sigma+\lambda) \hat{y}_t + \frac{(1-\rho)(2-\beta\rho)}{\rho} \frac{\tau}{1+\tau} \hat{\tau}_t + \beta \hat{\pi}_{P,t+1}. \tag{A.34}$$

Table 1. Benchmark parameter

β	σ	λ	θ	ω	ρ	ϕ_y	ϕ_π	τ
0.99	1	1	4	0.5	0.43	0.5	1.5	0.08

Figure 1. Policy interest rate and consumption tax rate in Japan

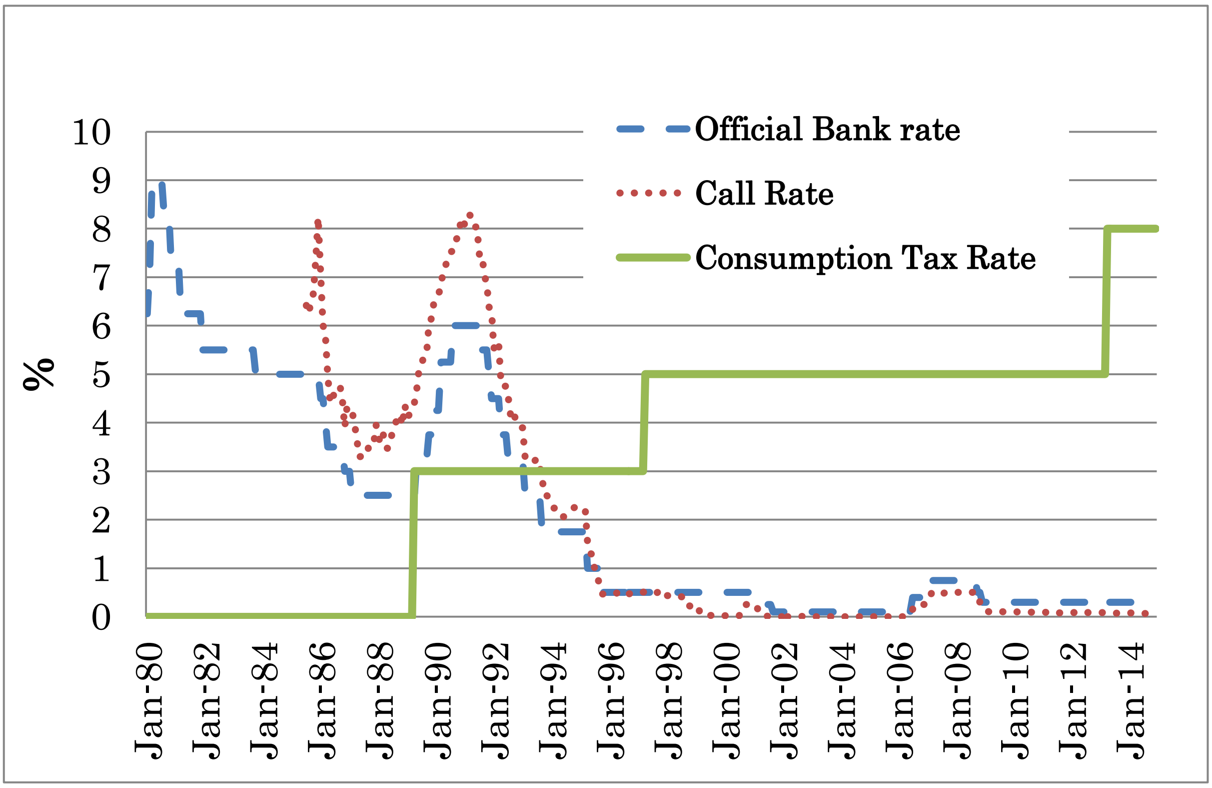


Figure 2. Policy interest rate and VAT rate in U.K.

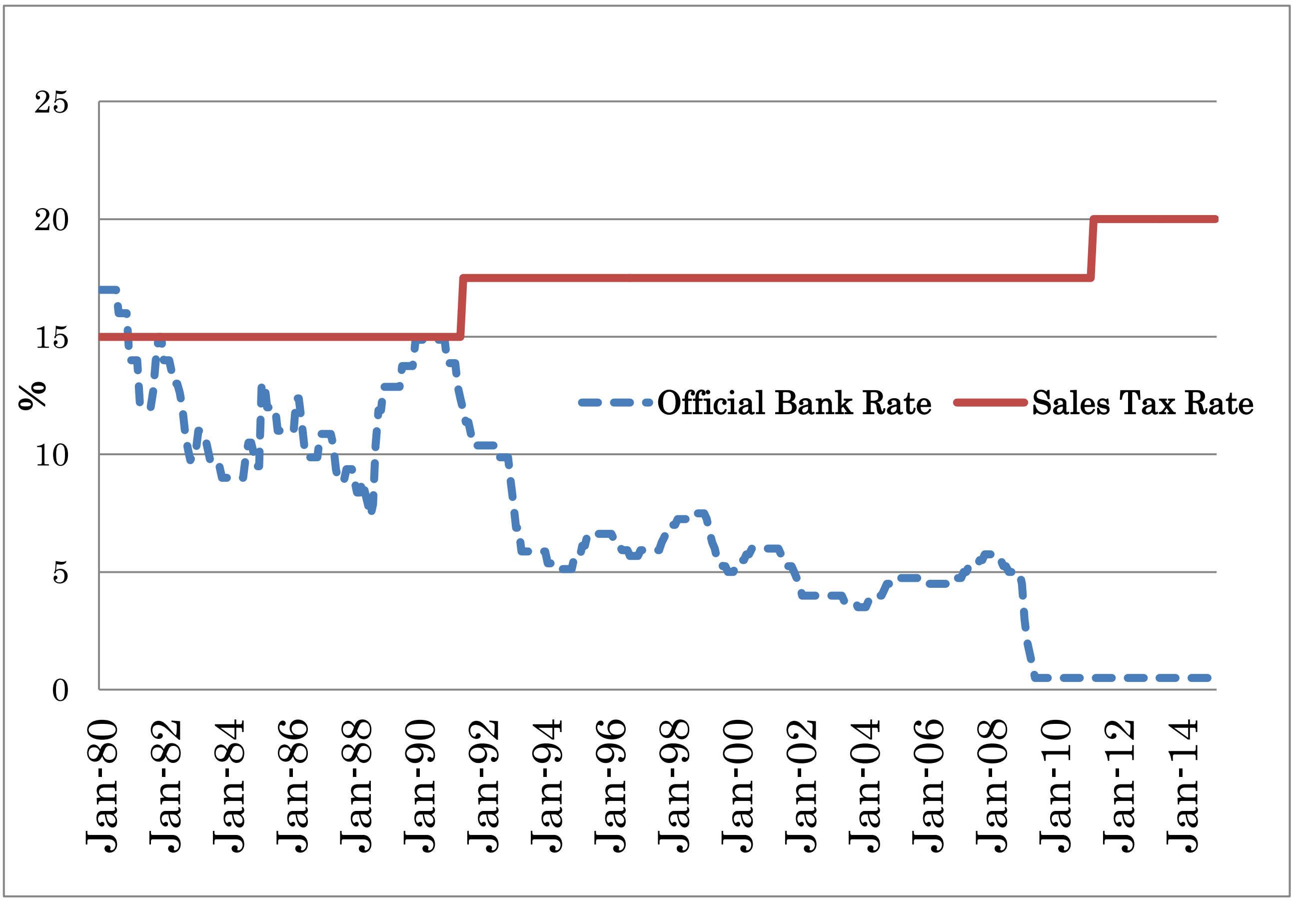


Figure 3. Growth rate (year on year) Consumer Price Index (CPI) and Producer Price Index (PPI) in Japan

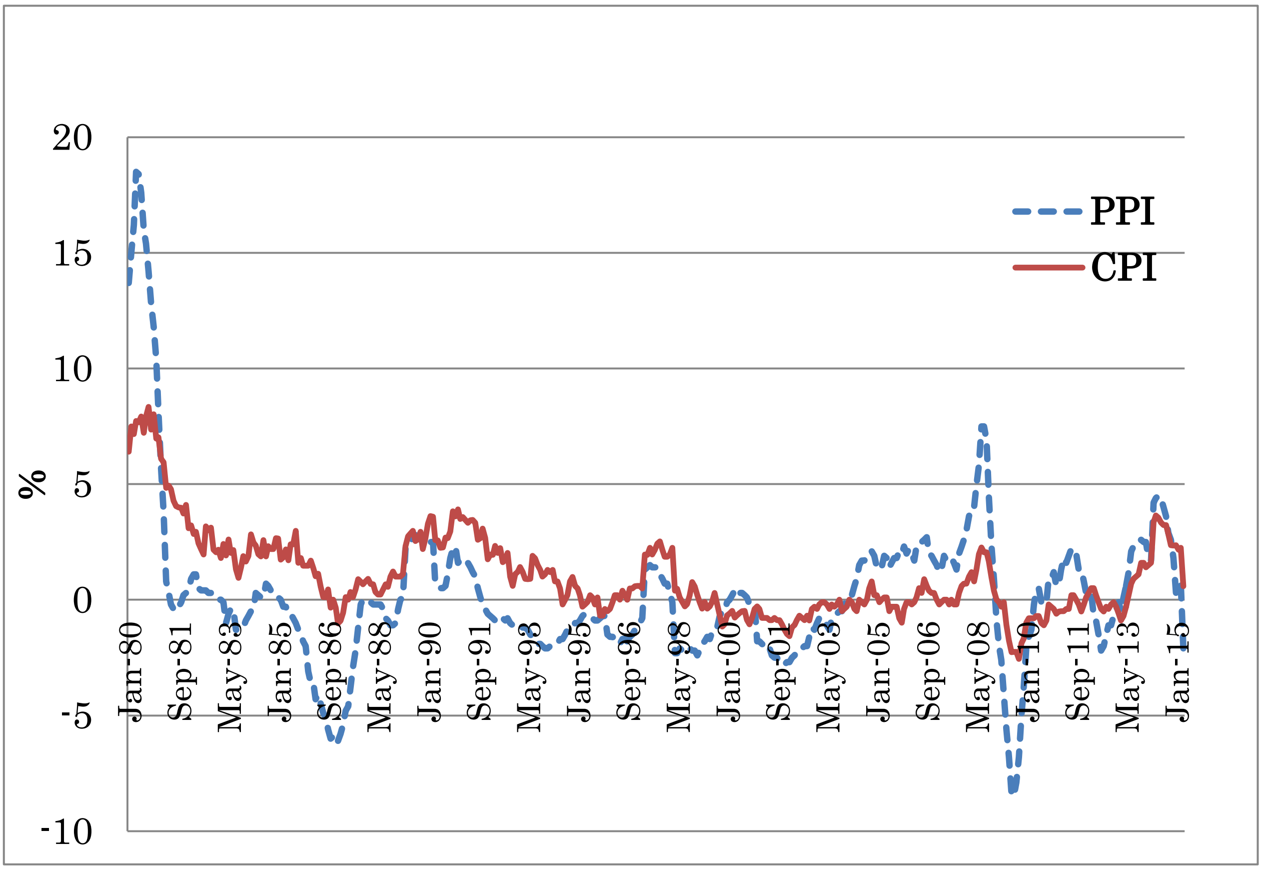


Figure 4. Impulse response of output to VAT increase shock under PPI and CPI target

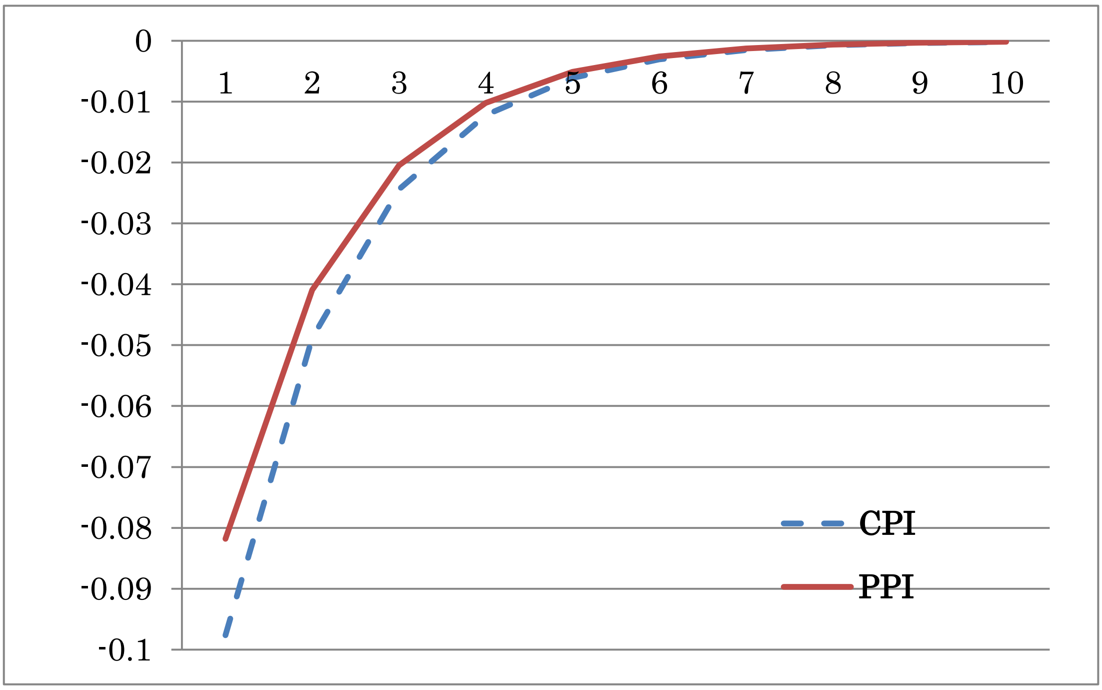


Figure 5. Impulse response of CPI inflation rate to VAT increase shock under PPI and CPI target

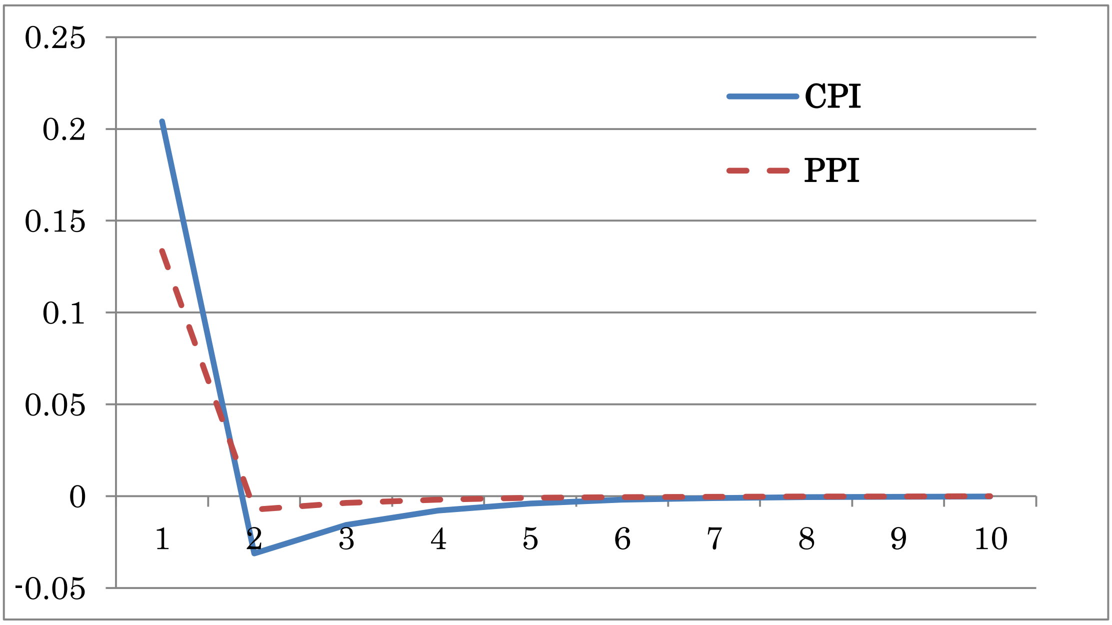
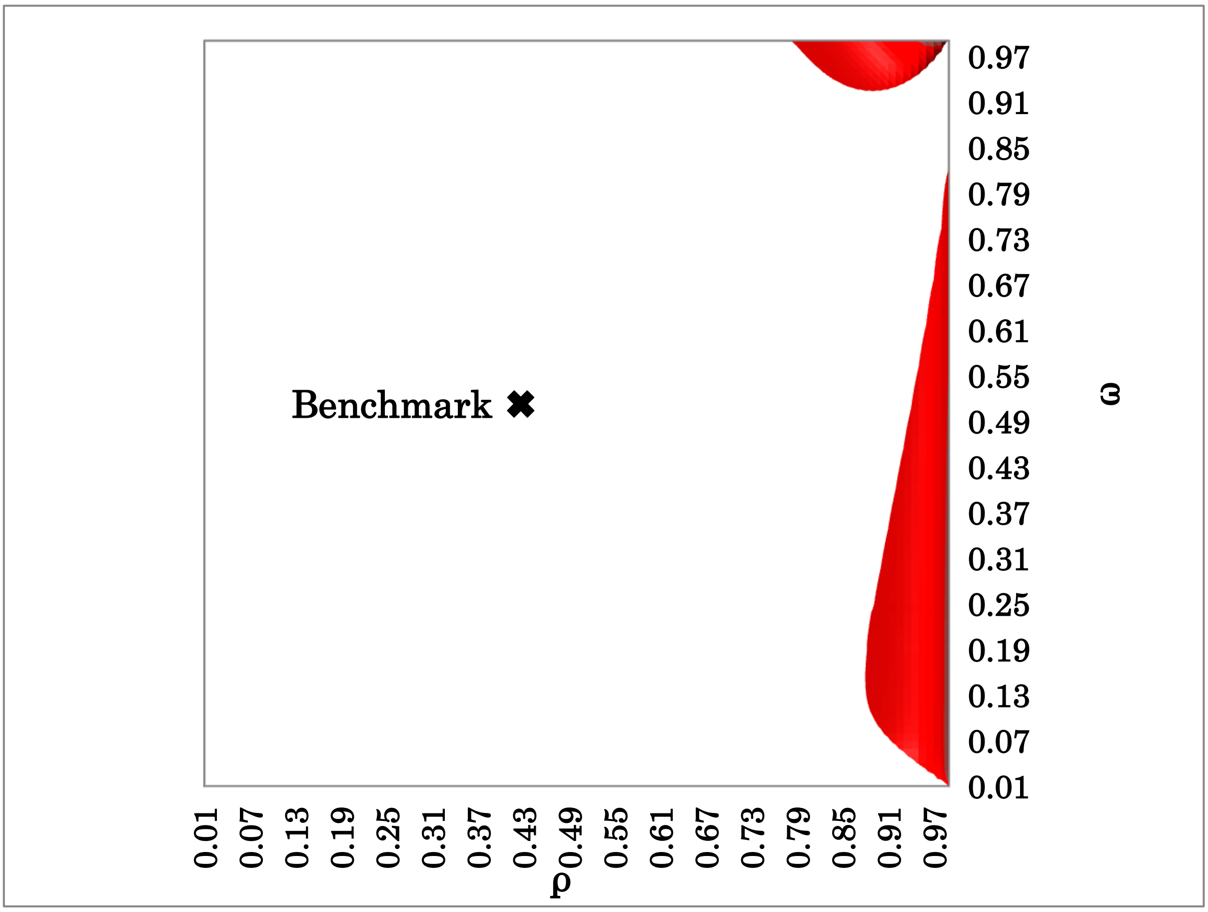


Figure 6. Welfare Comparison with two (CPI vs. PPI) price target policies



Note:

Region painted by red is that of the CPI targeting dominance (i.e. $U_{0,CT} > U_{0,PT}$).

Figure 7. Impulse response of output at period 1 in each ρ

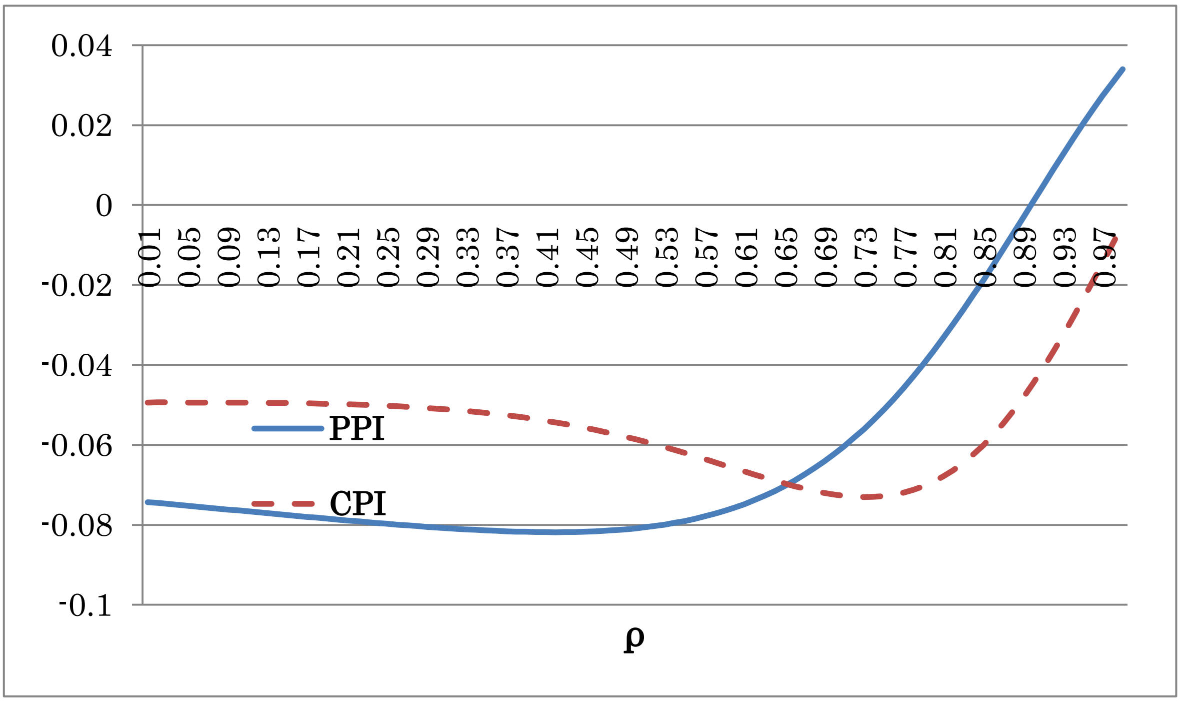


Figure 8. Impulse response of CPI inflation rate at period 1 in each ρ

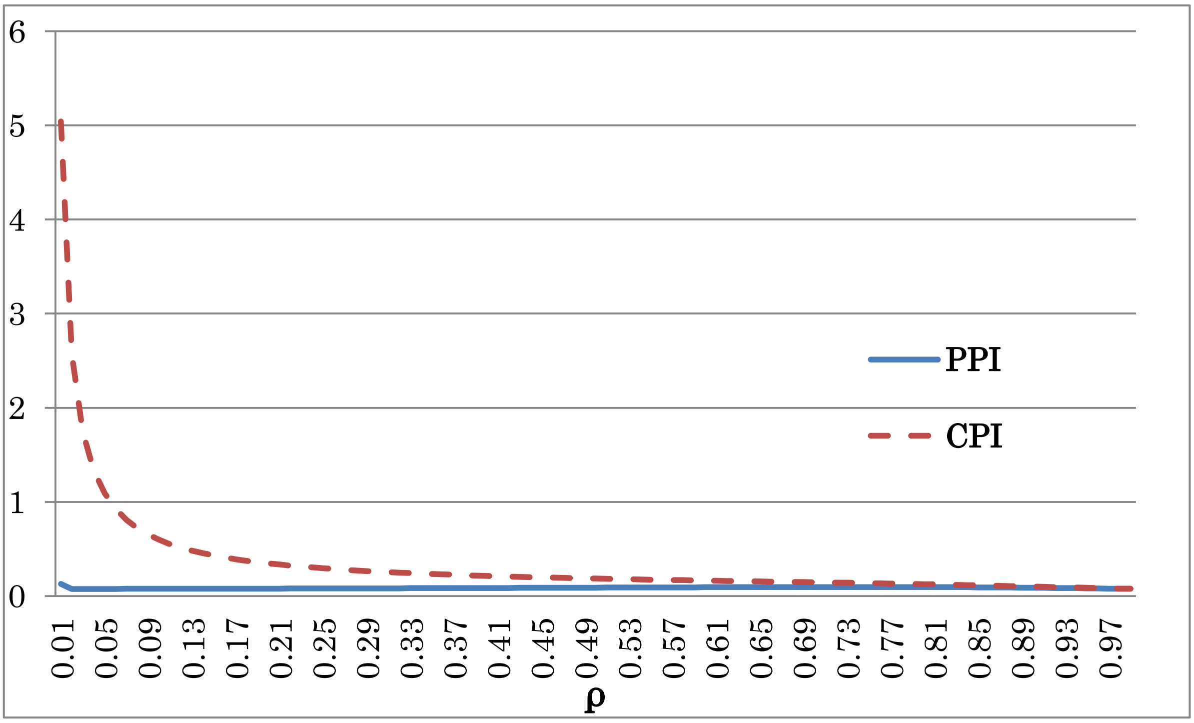


Figure 9. Impulse response of output to VAT increase shock under PPI and CPI target (ρ =0.99)

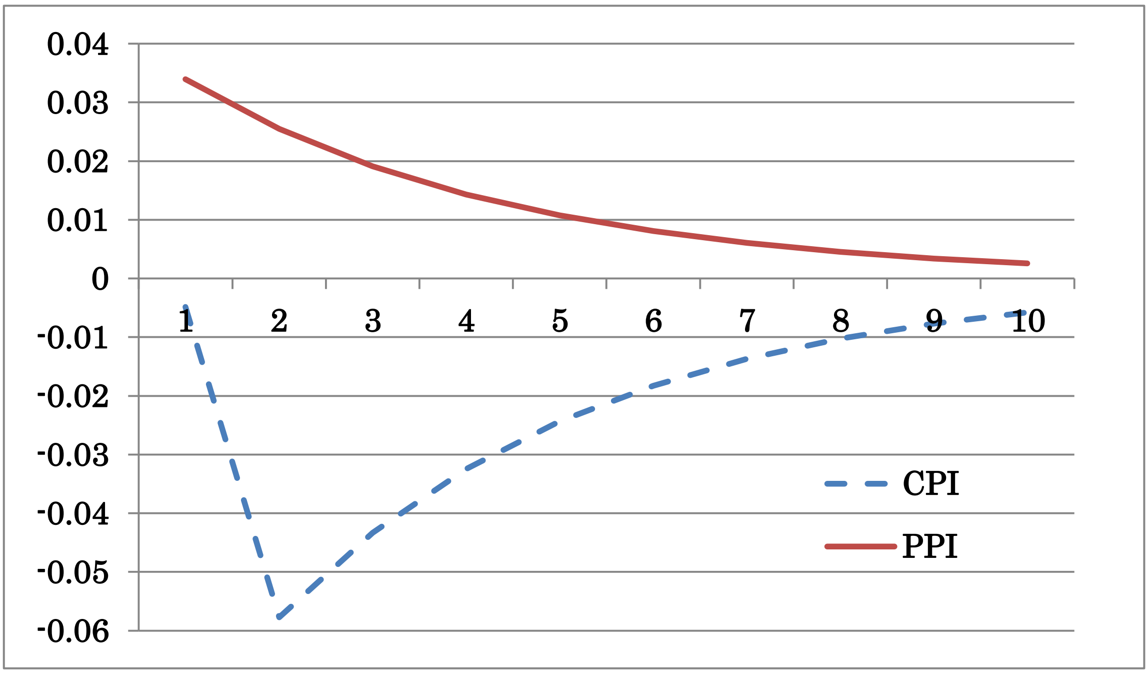


Figure 10. Impulse response of CPI inflation rate to VAT increase shock under PPI and CPI target (ρ =0.99)

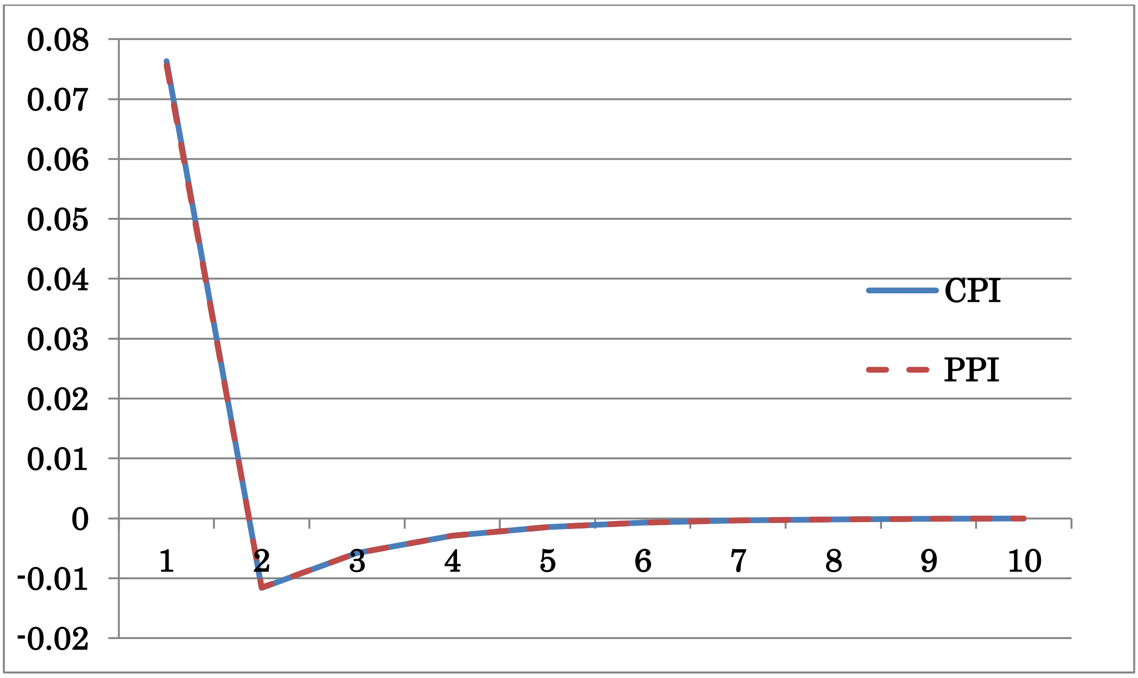


Figure 11. Impulse response of output to anticipated VAT increase shock under PPI and CPI target

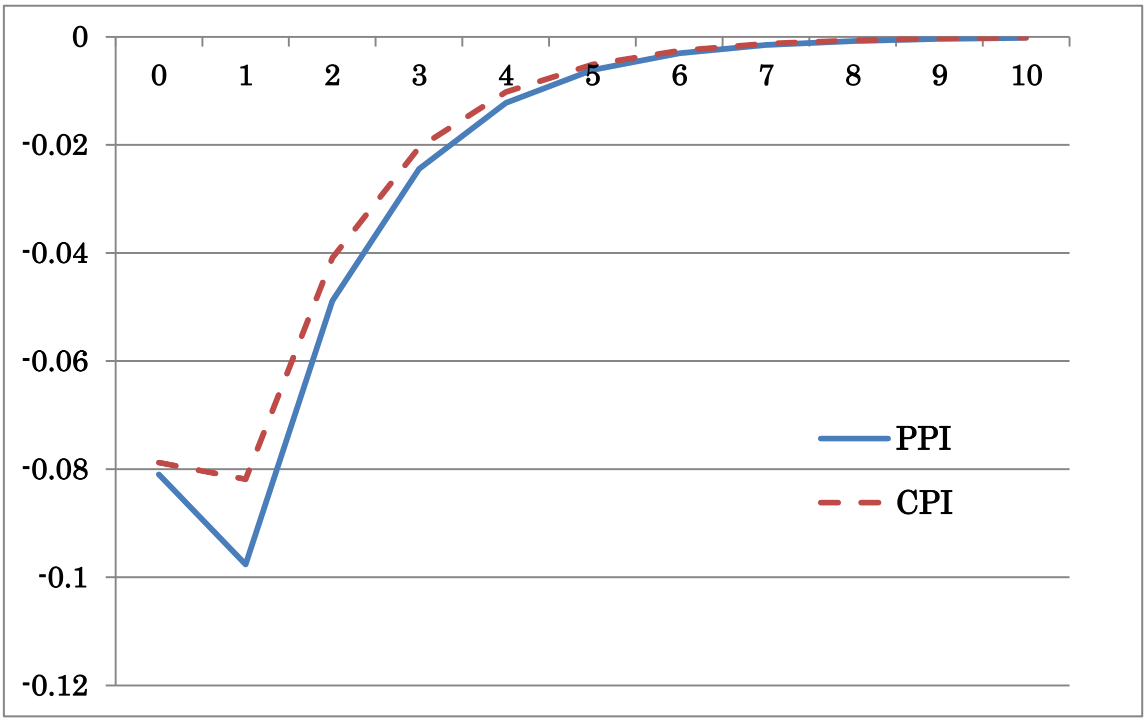


Figure 12. Impulse response of CPI inflation rate to anticipated VAT increase shock under PPI and CPI target