An Industrial-Organization Approach to Money and Banking*

Hiroshi Gunji†   Kenji Miyazaki
Daito Bunka University   Hosei University

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Abstract

In this paper, we study the effect of conventional interest rate policy, quantitative easing and the reserve accounts' interest rate on the money stock in an industrial-organization model of the banking industry with money creation. Our main findings are as follows. First, under a plausible setting of the parameters, the model with money creation supports the liquidity puzzle, in which tight monetary policy increases the money stock. Second, quantitative monetary easing has a similar effect. Third, the negative interest rate policy on reserves has a negative effect on the money stock.

JEL classification: E51, E52, G21.

1 Introduction

Since the 2000s, many central banks have conducted unconventional monetary policy. In particular, the Bank of Japan, the Federal Reserve Bank and the Bank of England adopted Quantitative Easing (QE), where the central bank purchased bonds to lower interest rates and/or increase the monetary base. Recently, the European Central Bank and the Bank of Japan set a negative interest rate on reserve accounts. This is called Negative Interest Rate Policy (NIRP).

In this paper, we use an industrial-organization model of the banking industry to approach the effect of conventional interest rate policy, QE, and NIRP on the money stock. In his seminal paper, Klein (1971) introduced an industrial-organizational model of the banking sector called the Monti–Klein model. Because it is quite tractable, many researchers have used this model: Pringle (1973), Towey (1974), Miller (1975), Dermine (1986), and Freixas and Rochet (2008). However, there is no research that studies the effects of conventional interest rate policy, QE, and NIRP on the money stock in the Monti–Klein model.

In their seminal work, Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) use DSGE models with financial intermediaries to show that QE plays an important role in the prevention of financial turmoil. In their model, financial intermediaries secure finance from households that they then provide as loans to firms. However, their models do not consider money creation. In fact, banks actually provide loans to both households and firms while taking deposits from

them at the same time. Banks do not need any resources to lend money. We now introduce money creation into our model to investigate the effect of monetary policy.

Our main contributions are as follows. First, the Monti–Klein model with money creation can give rise to the liquidity puzzle under a particular condition. When banks face an increase in money market interest rates, they finance their lending from an alternative instrument, i.e., deposits. Therefore, the money stock, which includes deposits, increases. Monetary theory suggests that a rise in the interest rate caused by monetary policy decreases the money stock. However, most of the empirical studies report the inverse of this, that is, a positive correlation between interest rates and the money stock. This is called the liquidity puzzle. A large number of researchers tried to resolve this puzzle by using alternative empirical methods.[1] Strongin (1995) and Christiano et al. (1996) find that, in general, a narrower definition of the money stock removes the puzzle. Moreover, Kelly et al. (2011) suggests that measurement error plays an important role in the liquidity puzzle. However, few theoretical studies capture the puzzle.

Second, we show that quantitative monetary easing in the Monti–Klein model has a similar effect. The condition under which the liquidity puzzle is observed in the model with quantitative easing is similar to that in the model with conventional monetary policy. In both cases, the liquidity effect depends on the ratio of borrowers' deposits to their loans.

Third, interest rates on reserves have a positive effect on the money stock. Because the interest on reserves benefits not only deposits but also loans to the banking sector, a rise in the rates on reserves increases the money stock, which consists of both deposits and loans. However, negative interest rates on reserves, as implemented by the Bank of Japan, has a negative effect on the money stock.

The remainder of this paper is as follows. In the next section, we introduce the conventional Monti–Klein model to study the effect of monetary policy on the money stock. We then introduce money creation and show that under a certain condition, the model with money creation results in the liquidity effect. In Sections 3 and 4, we consider the effect of quantitative easing and the interest rate on reserves, respectively. In the final section, we conclude.

2   The effect of the money market interest rate

2.1   The Monti–Klein model

Suppose that there are N banks in this economy. They supply loans, L_i, and money market lending, M_i, while they borrow from deposits, D_i $(i = 1, \ldots, N)$. Bank i maximizes its profits

$$\pi_i(L_i, M_i, D_i) = r_l(L)L_i + rM_i - r_d(D)D_i, \tag{1}$$

subject to the balance sheet

$$L_i + M_i + \alpha D_i = D_i, \tag{2}$$

where $r_l(L)$ is the interest rate on loans as the inverse demand function for loans ($r_l' < 0$), $r_d(D)$ is the interest rate on deposits as the inverse supply function for deposits ($r_d' > 0$), r is the rate on the money market as an instrument of monetary policy, and α is the ratio of reserves to deposits. Because there are non-bank financial institutions, the sum of M_i is not necessarily zero. α consists of not only the reserve requirement rate but also other demands for central bank deposits, because in some countries, e.g., the UK, there is no reserve requirement on bank deposits, but commercial banks are willing to demand reserves in order to settle inter-bank transactions. So, we assume $\alpha \in (0, 1]$. The left-hand side of this equation presents the asset

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[1]For earlier studies, see Melvin (1983) and Leeper and Gordon (1992).

side: bank loans, money market lending and reserves, while the right-hand side means the debt side. Combining these equations, we have

$$\pi_i(L_i, D_i) = [r_l(L) - r]L_i + [r(1 - \alpha) - r_d(D)]D_i. \tag{3}$$

For the non-banking sector that includes households and firms, we assume the inverse demand function for loans and the inverse supply function for deposits is

$$r_l(L) = a_l - b_l L \qquad \text{and} \qquad r_d(D) = a_d + b_d D, \tag{4}$$

where $a_l, b_l, a_d, b_d > 0$, $L = \sum_{i=1}^{N} L_i$, and $D = \sum_{i=1}^{N} D_i$. From the first order conditions, we obtain the Cournot–Nash equilibrium,

$$L_i = \frac{a_l - r}{(N + 1)b_l} \text{ and } D_i = \frac{r(1 - \alpha) - a_d}{(N + 1)b_d}, \tag{5}$$

for $i = 1, \ldots, N$. The effects of the money market interest rate are[2]

$$\frac{\partial L_i}{\partial r} = -\frac{1}{(N + 1)b_l} \quad \text{and} \quad \frac{\partial D_i}{\partial r} = \frac{1 - \alpha}{(N + 1)b_d}. \tag{6}$$

This model implicitly assumes that banks provide their loans L as cash because bank loans do not correspond to their liabilities, i.e., deposits, in the balance sheet. As we will study in the next subsection, in practice, if banks lend to firms and households, a part of L_i corresponds to the deposits. However, the conventional banking sector model does not consider such a situation. Therefore, the money stock in this economy consists of cash (loans) and deposits,

$$M^s = L + D = \sum_{i=1}^{N}(L_i + D_i). \tag{7}$$

In equilibrium, the effect of the money market interest rate on the money stock is

$$\frac{\partial M^s}{\partial r} = \frac{N}{N + 1} \left(\frac{1 - \alpha}{b_d} - \frac{1}{b_l} \right). \tag{8}$$

Proposition 1 *In the Monti–Klein model, the effect of the money market interest rates on the money stock is*

$$\frac{\partial M^s}{\partial r} \begin{cases} > 0 & \text{if } \frac{b_l}{b_d} > \frac{1}{1 - \alpha} \\ = 0 & \text{if } \frac{b_l}{b_d} = \frac{1}{1 - \alpha} \\ < 0 & \text{otherwise.} \end{cases} \tag{9}$$

This result can explain the liquidity puzzle. Standard monetary theory, e.g., the IS-LM model, neoclassical growth model with money-in-the-utility or cash-in-advance constraint and so on, suggests that a tight monetary policy decreases the money stock. However, many empirical studies report the opposite effect, especially for broad money. In the Monti–Klein model, a rise in money market interest rates leads to an incentive of banks to lend to the open market, financing these loans from deposits. Therefore, if $b_l/b_d > 1/(1 - \alpha)$, a tight monetary policy *increases* the money stock.

In fact, however, depositors do not react so much to a change in deposit rates, whereas borrowers react strongly to a change in lending rates. That is, b_d is probably greater than b_l. If so, b_l/b_d is relatively small. Therefore, in this model, it is likely that an increase in money market rates decreases the money stock.

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[2]Individual bank loans L_i decrease as N increases. However, aggregate bank loans, $L = \sum_{i=1}^{N} L_i$, increase as N increases. This is partly consistent with the empirical fact shown by Gunji et al. (2009).

2.2 The Monti–Klein model with money creation

As shown in the previous section, the Monti–Klein model can demonstrate the liquidity puzzle. However, the result depends on b_l/b_d, which is usually small in practice. Moreover, borrowers do not, in fact, withdraw their deposits very much. To resolve the shortcomings of the model in the previous subsection, we introduce money creation into the model.

We divide deposits into saving accounts, D_{0i}, and checking accounts, D_{li}. The profit of bank i is

$$\pi_i = r_l(L)L_i + rM_i - r_d(D_0)D_{0i}. \tag{10}$$

where $D_i = D_{0i} + D_{li}$ is the total deposits of bank i and $D_0 = \sum_{i=1}^{N} D_{0i}$. The balance sheet of bank i is

$$L_i + M_i + \alpha' D_{0i} = D_{0i} + D_{li}. \tag{11}$$

In this case, the reserve ratio α' is not identified, because banks can choose both α' and the amount of the saving deposit D_{0i}. Therefore, banks face the other constraint $\alpha' D_{0i} = \alpha(D_{0i} + D_{li})$, in which banks choose α' and D_{0i} so as to satisfy their demand for reserves $\alpha(D_{0i} + D_{li})$. This constraint yields

$$L_i + M_i + \alpha(D_{0i} + D_{li}) = D_{0i} + D_{li}. \tag{12}$$

Banks lend to borrowers as checking accounts and borrowers partly draw down their account, that is,

$$D_{li} = \theta_i L_i, \tag{13}$$

where $\theta_i \in [0,1]$ is the ratio of checking accounts to loans and $1 - \theta_i$ is the ratio of withdrawals from deposits, i.e. cash, to loans. Banks may provide their loans to a number of borrowers, which individually withdraw different amounts of cash from their deposits. At the same time, the borrowers earn revenue and repay the banks. Consequently, θ_i averaged among borrowers would usually be positive. In practice, θ_i is close to one in the modern economy. For simplicity, we assume that all banks face the same θ_i condition, that is, $\theta_i = \theta$ for all i. It is important to note that banks can make loans without the central bank's injections from the monetary base because banks are required to finance L_i, which can be satisfied, for example, as checking accounts of θL_i and saving accounts of $(1 - \theta)L_i$. In other words, money creation does not initially require base money.

Substituting (13) into (12), we obtain

$$M_i = (1 - \alpha)D_{0i} - [1 - \theta(1 - \alpha)]L_i. \tag{14}$$

The second term of the right-hand side implies that to lend L_i dollars, bank i has to borrow $1 - \theta(1 - \alpha)$ dollars from the money market, instead of L_i dollars, because the remainder of checking accounts $(1 - \alpha)D_{li}$ is also available for loans. So the optimization problem of bank i is to maximize

$$\pi_i = \{r_l(L) - r[1 - \theta(1 - \alpha)]\}L_i + [r(1 - \alpha) - r_d(D_0)]D_{0i}. \tag{15}$$

From the first order conditions, we obtain

$$L_i = \frac{a_l - r[1 - \theta(1 - \alpha)]}{(N+1)b_l} \text{ and } D_{0i} = \frac{r(1 - \alpha) - a_d}{(N+1)b_d}. \tag{16}$$

In this economy, bank loans are greater by $r\theta(1 - \alpha)/(N + 1)b_l$ than in the Monti–Klein model. Because total deposits are saving accounts plus checking accounts,

$$D_i = D_{0i} + D_{li} = D_{0i} + \theta L_i. \tag{17}$$

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The effect of the money market interest rate on bank loans is

$$\frac{\partial L_i}{\partial r} = -\frac{1 - \theta(1 - \alpha)}{(N + 1)b_l}. \tag{18}$$

This derivative consists of two terms. The first is the same as the Monti–Klein model, $-1/(N+1)b_l$, in which a rise in r increases the financing cost from the money market. The second is caused by money creation, in which a rise in r increases interest earnings by lending checking accounts to the money market, that is $\theta(1 - \alpha)/(N + 1)b_l$. Therefore, with money creation, the effect of money market interest rates on bank loans is equal to or smaller than that without money creation.

On the other hand, the effect of the money market interest rate on total deposits is

$$\frac{\partial D_i}{\partial r} = \frac{\partial D_{0i}}{\partial r} + \theta \frac{\partial L_i}{\partial r} = \frac{1 - \alpha}{(N + 1)b_d} - \frac{\theta[1 - \theta(1 - \alpha)]}{(N + 1)b_l}. \tag{19}$$

This suggests that $\partial D_i/\partial r$ in the model with money creation is not only smaller than that without money creation, but can also still be positive under a certain condition.

Lemma 1 *In the Monti–Klein model with money creation, the effect of money market interest rates on bank loans is negative and (i) if $b_l/b_d > 1/[4(1 - \alpha)^2]$, then $\partial D_i/\partial r > 0$. (ii) If $b_l/b_d = 1/[4(1 - \alpha)^2]$, then $\partial D_i/\partial r \geq 0$. (iii) If $b_l/b_d < 1/[4(1 - \alpha)^2]$, then*

$$\frac{\partial D_i}{\partial r} \begin{cases} > 0 & \text{if } 0 \leq \theta < \underline{\theta} \text{ or } \overline{\theta} < \theta \leq 1, \\ = 0 & \text{if } \theta = \underline{\theta} \text{ or } \theta = \overline{\theta}, \\ < 0 & \text{if } \underline{\theta} < \theta < \overline{\theta}, \end{cases} \tag{20}$$

where $\underline{\theta} = \frac{b_d - \sqrt{b_d[b_d - 4(1-\alpha)^2 b_l]}}{2(1-\alpha)b_d}$ and $\overline{\theta} = \frac{b_d + \sqrt{b_d[b_d - 4(1-\alpha)^2 b_l]}}{2(1-\alpha)b_d}$.

Proof. Equation (19) depends on θ. Define $f(\theta) = (1 - \alpha)/b_d - (1/b_l)\theta + [(1 - \alpha)/b_l]\theta^2$. The discriminant of $f(\theta) = 0$ is $\Delta = 1/b_l^2 - 4(1 - \alpha)^2/(b_l b_d)$. Since $f''(\theta) = 2(1 - \alpha)/b_l > 0$, $\Delta < 0$ $\Leftrightarrow f(\theta) > 0$ and $\Delta = 0 \Leftrightarrow f(\theta) \geq 0$. Iff $\Delta > 0$, then $f(\theta) = 0$ has two roots, $\underline{\theta}$ and $\overline{\theta}$. $\square$

Figure 1 shows an example of this result. The parameters are set to be $N = 10$, $\alpha = 0.01$, $b_d = 1$, and $b_l = \{2, 0.2551, 0.1\}$, where $1/[4(1 - \alpha)^2] \simeq 0.2551$. The vertical axis is $\partial D_i/\partial r$ and the horizontal axis is θ. In the case of $\theta = 0$, which corresponds to the Monti–Klein model without money creation, the effect of r is always positive. However, as θ approaches 0.2551, the effect decreases. The reason is that higher θ leads to a rise in bank loans and increases the effect of r on L_i that has a negative sign. On the other hand, it rises as θ goes from 0.2551 to unity, because a rise in θ brings about an increase in checking accounts, D_0, and yields more earnings from the money market.

In this economy, the money stock consists of cash currency and deposit currency,

$$M^s = \underbrace{(1 - \theta) \sum_{i=1}^{N} L_i}_{\text{cash currency}} + \underbrace{\sum_{i=1}^{N} (D_{0i} + \theta L_i)}_{\text{deposit currency}} = D_0 + L. \tag{21}$$

It is important to note that in this economy, cash and checking deposits are created by loans. As a result, we define the money stock as consisting of saving deposits and loans, but also including cash. Therefore, we obtain the effect of the short-term interest rate on the money stock,

$$\frac{\partial M^s}{\partial r} = -\frac{N}{N + 1} \left[\frac{1 - \theta(1 - \alpha)}{b_l} - \frac{1 - \alpha}{b_d} \right]. \tag{22}$$

Proposition 2 *In the Monti–Klein model with money creation, the effect of money market interest rates on the money stock is*

$$\frac{\partial M^s}{\partial r} \begin{cases} > 0 & if \ \frac{b_l}{b_d} > \frac{1-\theta(1-\alpha)}{1-\alpha} \\ = 0 & if \ \frac{b_l}{b_d} = \frac{1-\theta(1-\alpha)}{1-\alpha} \\ < 0 & otherwise. \end{cases} \tag{23}$$

Figure 2 demonstrates the impact of money-market interest rates on the money stock. In the case of the solid line, where the slope of the interest rate curve on loans, b_l, is relatively greater than that on deposits, b_d, the effect of the money market rate on the money stock is positive in spite of θ. However, as the ratio of b_l to b_d decreases, the effect is reflected in θ. Therefore, the liquidity puzzle is observed when the ratio, b_l/b_d is sufficiently large. Suppose, on the other hand, that if θ is sufficiently large, e.g., $\theta = 1$. Then, the right-hand side of the condition in Proposition 2 becomes $\alpha/(1 - \alpha)$. In practice, α is quite small, so is $\alpha/(1 - \alpha)$. For example, when $\alpha = 0.01$, we obtain $\alpha/(1 - \alpha) \simeq 0.01$. In this case, the model generates the liquidity puzzle unless b_l/b_d is close to zero.

The reason is that with money creation, a rise in the money market interest rate induces banks to decrease loans and the effect is large when the slope of the interest rate curve for loans b_l is small. Hence, the liquidity effect depends on the indirect effect of r on L.

2.3 Calibrating θ

The liquidity effect in the Monti–Klein model critically depends on the ratio of the deposits of borrowers to loans, θ. The money stock consists of cash currency and deposit currency, which are related to bank loans. Therefore, it is important to investigate how the borrowers withdraw or leave their deposits. In this section, we calibrate θ from the data.

By definition, we obtain $\theta = D_l/L$, but D_l is not the same as actual checking accounts data. Although D_l is provided when banks lend L to borrowers, we cannot identify how much deposits rise with an increase of bank loans. Therefore, we calibrate θ using an alternative method. First, we define the ratio of loans to deposits, $\beta_0 \equiv L/D$, and, because $D = D_0 + \theta L$, we have

$$\theta = \frac{L - \beta_0 D_0}{\beta_0 L}.$$

Second, the money multiplier is defined as $\beta_1 \equiv M^s/M^b$. From the definitions of M^s and M^b, we obtain

$$D_0 = \frac{\beta_1 - 1}{1 - \alpha\beta_1} L - \frac{1 - \alpha}{1 - \alpha\beta_1} L\beta_1\theta.$$

Therefore, we have

$$\theta = \frac{(1 - \alpha\beta_1)/\beta_0 - \beta_0(\beta_1 - 1)}{1 - \alpha\beta_1 - (1 - \alpha)\beta_0\beta_1}.$$

We use Japanese data from the website of the Bank of Japan to estimate θ. The sample period is from 2003 to 2014. The monetary base is M^b, M3 is M^s, the amount of loans of domestic banks is L, and the amount of deposits of domestic banks is D.

The result is shown in Figure 3. Although the estimate of θ moves around 0.85, it falls sharply in 2013, when the Bank of Japan introduced Quantitative and Qualitative Monetary Easing (QQE). Therefore, in the period of conventional monetary policy, θ is quite high. This implies that in the normal period, the liquidity period is prone to be observed.

3 Quantitative monetary easing and the interest rate on reserve accounts in the Monti–Klein model

If the money market rate of interest reaches the zero lower bound, the central bank cannot decrease the rate anymore. Alternatively, some central banks have purchased long-term government bonds and/or other risky bonds from the financial market in order to affect the yield curve directly. This is called quantitative monetary easing. In this section, we study the effects of bond purchases by the central bank using the model with money creation.

The central bank can also use interest on reserve balances as an instrument of monetary policy. In particular, the Bank of Japan has conducted a NIRP, which is a negative rate of interest on excess reserve balances, since February 2016. However, the effect is not well known. So, in this section, we use the Monti–Klein model to investigate the effect of the interest rate on reserve balances. Because the model without money creation has a similar result to that below, we present only the model with money creation.

For simplicity, we ignore money market lending and introduce bonds into the model. The profit of bank i is

$$\pi_i = r_l(L)L_i + r_b(B)B_i - r_d(D_0)D_{0i} + r_{CB}\alpha(D_{0i} + D_{li}). \tag{24}$$

where $B = B_{CB} + \sum_{i=1}^{N} B_i$, B_i is the bond holdings of bank $i = 1, \ldots, N$ or the central bank $i = CB$, $r_b(B)$ is the yield of bonds, and r_{CB} is the interest rate on reserves. Although in practice, the Bank of Japan applies NIRP only on excess reserves, we assume for simplicity that the interest rate applies to total reserves. We assume the bond yield is

$$r_b(B) = a_b - b_b B, \tag{25}$$

where $a_b, b_b > 0$. The central bank first purchases bonds from non-bank institutions and pays into the reserves of a bank that holds the institutions' deposit accounts. This operation leads to increases both in the asset side (reserves) and in the liability side (deposits of non-bank institutions) of the bank balance sheet. The balance sheet of bank i is

$$L_i + B_i + \alpha D_{0i} + R_{Ni} = D_{0i} + D_{li} + D_{Ni},$$

where $D_{Ni} = B_{CB}$ is new deposits that non-bank institutions obtain by selling their bonds to the central bank and $R_{Ni} = B_{CB}$ is new reserves. Therefore, it has no effect on the constraint of banks and we have[3]

$$L_i + B_i + \alpha(D_{0i} + D_{li}) = D_{0i} + D_{li}. \tag{26}$$

Substituting (13) into (26), we obtain

$$B_i = (1 - \alpha)D_{0i} - [1 - \theta(1 - \alpha)]L_i. \tag{27}$$

The first order conditions are

$$L_i = \frac{\Phi_1}{2\Phi_0} - \frac{1}{2}\sum_{j \neq i} L_j + \frac{\Phi_2}{2\Phi_0}(D_{0i} + D_0), \tag{28}$$

$$D_{0i} = \frac{\Psi_1}{2\Psi_0} - \frac{1}{2}\sum_{j \neq i} D_{0j} + \frac{\Phi_2}{2\Psi_0}(L_i + L). \tag{29}$$

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[3]This assumption means that banks hold excess reserves. Even if, however, banks do not hold any excess reserves, that is, reserves$= \alpha(D_{0i} + D_{li} + D_{Ni})$, we obtain exactly the same condition as Proposition 3.

where

$$\Phi_0 = b_l + b_b[1 - \theta(1 - \alpha)]^2, \tag{30}$$

$$\Phi_1 = a_l - (a_b - b_b B_{CB})[1 - \theta(1 - \alpha)] + r_{CB}\alpha\theta, \tag{31}$$

$$\Phi_2 = b_b(1 - \alpha)[1 - \theta(1 - \alpha)], \tag{32}$$

$$\Psi_0 = b_d + b_b(1 - \alpha)^2, \tag{33}$$

$$\Psi_1 = [a_b - b_b B_{CB}](1 - \alpha) - a_d + r_{CB}\alpha. \tag{34}$$

From these conditions, we have

$$L_i = \frac{\Phi_1\Psi_0 + \Phi_2\Psi_1}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]}, \tag{35}$$

$$D_{0i} = \frac{\Phi_0\Psi_1 + \Phi_1\Phi_2}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]}. \tag{36}$$

Proposition 3 *In the Monti–Klein model with money creation, the effect of purchasing bonds by the central bank on the money stock is*

$$\frac{\partial M^s}{\partial B_{CB}} \begin{cases} < 0 & \text{if } \frac{b_l}{b_d} > \frac{1 - \theta(1 - \alpha)}{1 - \alpha} \\ = 0 & \text{if } \frac{b_l}{b_d} = \frac{1 - \theta(1 - \alpha)}{1 - \alpha} \\ > 0 & \text{otherwise.} \end{cases} \tag{37}$$

Proof. See the Appendix.

From the Appendix, the effects of quantitative monetary easing on the money stock is

$$\frac{\partial M^s}{\partial B_{CB}} = \frac{N}{N + 1}\left(\frac{1 - \theta(1 - \alpha)}{b_l} - \frac{1 - \alpha}{b_d}\right)\left(\frac{1}{b_b} + \frac{[1 - \theta(1 - \alpha)]^2}{b_l} + \frac{(1 - \alpha)^2}{b_d}\right)^{-1}. \tag{38}$$

This formula is similar to Eq. (22) except for the second term in the reciprocal. Figure 4 demonstrates Eq. (38). We set $b_b = 2$ and the other parameters are the same as those in the previous section. In general, a rise in B_{CB} decreases its interest rate b_b, but does not necessarily increase M^s. As shown in the previous subsection, the smaller the ratio b_l/b_d, the greater $\partial M^s/\partial B_{CB}$. If, however, θ is sufficiently large, the derivative tends to be negative, that is, monetary policy easing decreases the money stock.

Next, we consider the effect of interest on reserves. The effect of the interest rate on reserve accounts is

$$\frac{\partial L_i}{\partial r_{CB}} = \frac{\alpha\theta}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]} \text{ and } \frac{\partial D_{0i}}{\partial r_{CB}} = \frac{\alpha}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]}. \tag{39}$$

Therefore, we obtain the effect of r_{CB} on the money stock,

$$\frac{\partial M^s}{\partial r_{CB}} = \frac{N}{N + 1}\frac{\alpha(1 + \theta)}{\Phi_0\Psi_0 - (\Phi_2)^2}. \tag{40}$$

Proposition 4 *In the Monti–Klein model, the effect of the interest rate on reserve balances on the money stock is positive.*

That is, NIRP *decreases* bank loans, deposits, and the money stock. The reason is that if the central bank reduces the interest rate on reserves, ceteris paribus, the cost of new bank loans increases because new loans create new deposits and requires new reserves. On the other hand, the sign of the effect of r_{CB} on B is ambiguous. The sign is positive (negative) if the effect of r_{CB} on D_{0i} (L_i) matters.

Is this effect stronger when the banking industry is more competitive? Mr. Haruhiko Kuroda, the Governer of the Bank of Japan, says:

> In fact, given the fierce competition in Japan's lending market, banks are unlikely to increase lending rates to offset the rise in costs through the negative interest rate policy. (March 7, 2016)

However, our model suggests

$$0 > \frac{dr_l}{dL} \left.\frac{\partial L}{\partial r_{CB}}\right|_{N=1} > \frac{dr_l}{dL} \left.\frac{\partial L}{\partial r_{CB}}\right|_{N\to\infty}.$$

In other words, the more competitive the banking industry, the higher the lending rates is from the effect of NIRP. This is because banks in a competitive market produce, that is, lend, more than those in a less competitive market. Hence, fierce competition among banks would lead to a tighter monetary policy for the economy.

4   Concluding remarks

We have studied monetary policy in the Monti–Klein model. The liquidity effect in the model with money creation depends on the ratio of the deposits of borrowers to their loans, θ. Under a certain condition, the model can yield the liquidity puzzle. This result is the same in the model with quantitative monetary easing: While the effect of the central bank's bond purchase on the money stock is positive in the normal model, the effect depends on θ in the model with money creation. Therefore, the liquidity puzzle is observed when θ is relatively high. On the other hand, the interest on reserves increases the money stock because it is an incentive for obtaining new deposits, which can be created by bank loans.

As you may notice, our study is of a partial equilibrium model. In our model, there is no interaction between loans, money market lending, and deposits, except the channel through the banking industry. This is a crucial limitation in our model. So, an important future task is to extend our model to a general equilibrium model.

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Appendix: Proof of Proposition 3

The effects of purchasing bonds by the central bank on bank loans and deposits are

$$\frac{\partial L_i}{\partial B_{CB}} = \frac{b_b[1 - \theta(1 - \alpha)]\Psi_0 - \Phi_2 b_b(1 - \alpha)}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]}, \tag{41}$$

$$\frac{\partial D_{0i}}{\partial B_{CB}} = \frac{\Phi_2 b_b[1 - \theta(1 - \alpha)] - \Phi_0 b_b(1 - \alpha)}{(N + 1)[\Phi_0\Psi_0 - (\Phi_2)^2]}. \tag{42}$$

The sign of the denominator, which is the same in both derivatives, is positive because

$$\Phi_0\Psi_0 - (\Phi_2)^2 = b_l b_d + b_l b_b(1 - \alpha)^2 + b_b b_d[1 - \theta(1 - \alpha)]^2 > 0. \tag{43}$$

On the other hand, the numerators are

$$b_b[1 - \theta(1 - \alpha)]\Psi_0 - \Phi_2 b_b(1 - \alpha) = b_d b_b[1 - \theta(1 - \alpha)] > 0, \tag{44}$$

$$\Phi_2 b_b[1 - \theta(1 - \alpha)] - \Phi_0 b_b(1 - \alpha) = -b_l b_b(1 - \alpha) < 0. \tag{45}$$

Therefore, $\partial L_i/\partial B_{CB} > 0$ and $\partial D_{0i}/\partial B_{CB} < 0$. Because the money stock in this economy is $M^s = \sum_{i=1}^{N}(D_{0i} + L_i)$, we obtain

$$\text{sgn}\left(\frac{\partial M^s}{\partial B_{CB}}\right) = \text{sgn}\left(b_d[1 - \theta(1 - \alpha)] - b_l(1 - \alpha)\right). \tag{46}$$

Hence, if (46) is positive, then $\partial M^s/\partial B_{CB}$ is also positive. $\square$
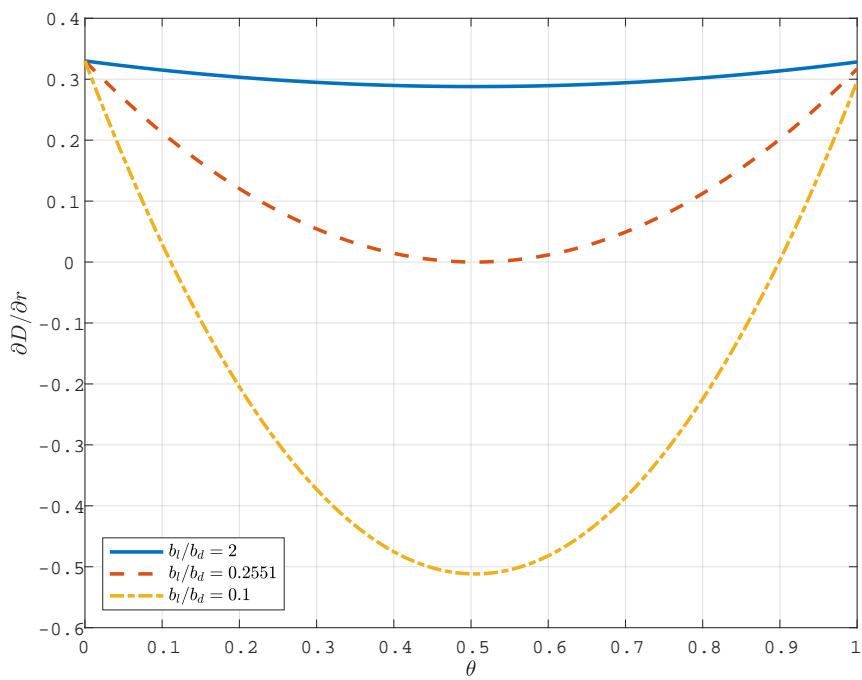
Figure 1: The impact of interest rate on deposits versus the ratio of checking accounts to loans
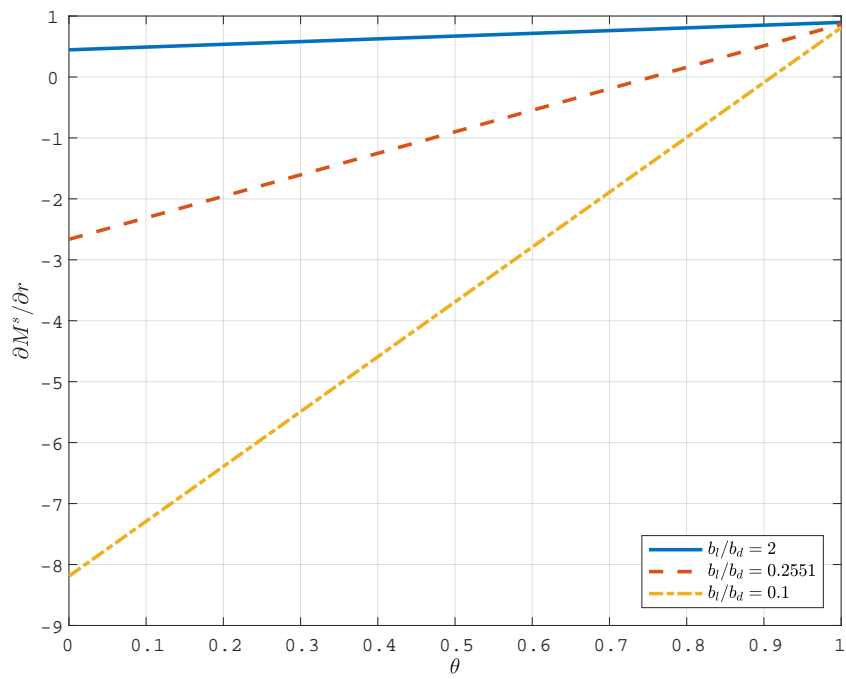
Figure 2: The impact of the interest rate on the money stock versus the ratio of checking accounts to loans
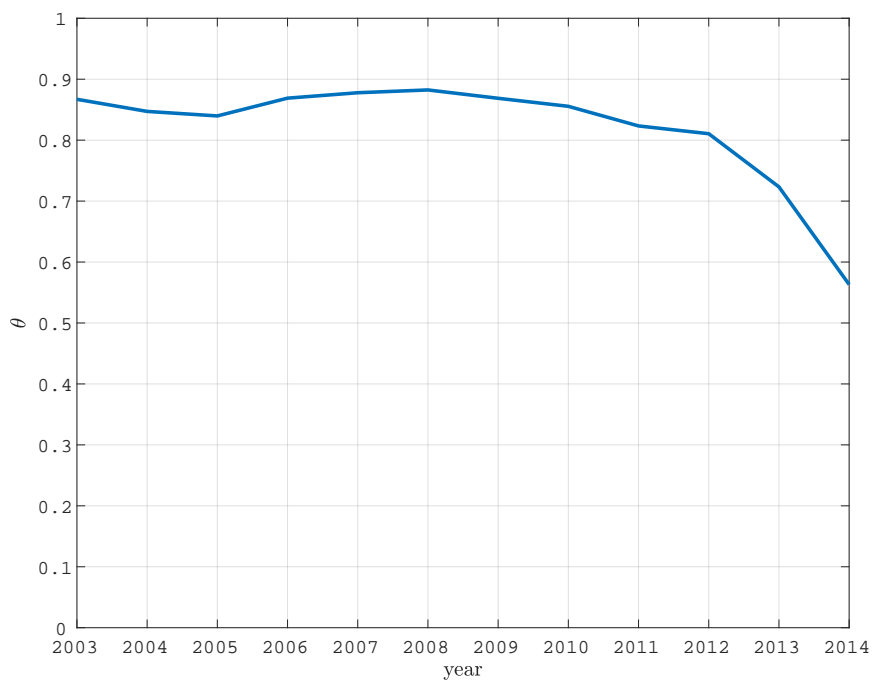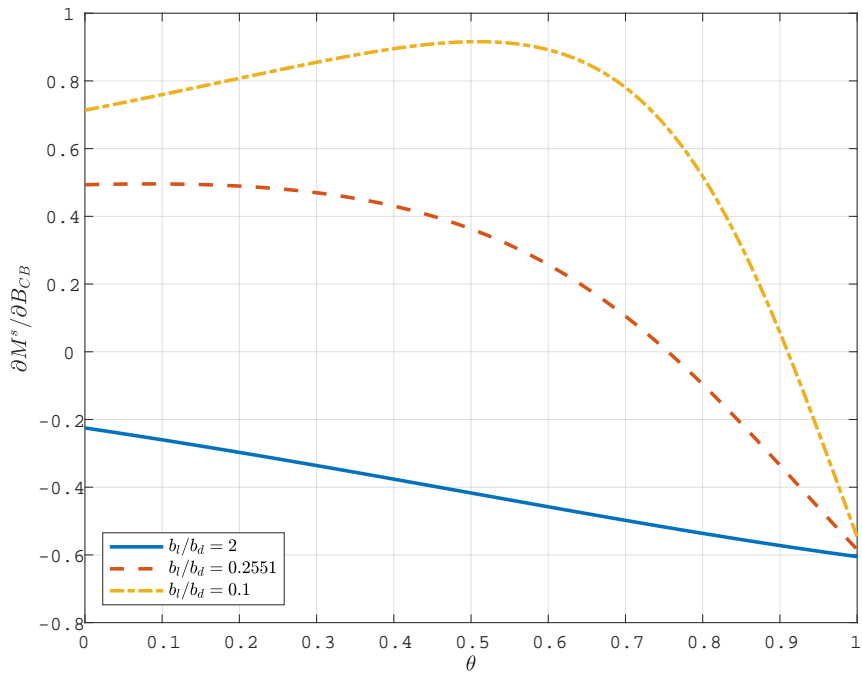
Figure 3: Calibrated theta

Figure 4: The impact of the central bank's bond purchasing on the money stock versus the ratio of checking accounts to loans