Okishio's Theory of Accumulation
In the Tradition of Heterodox Economics

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1. Introduction

About thirty years, which is equal to one generation, has passed since the Golden age of advanced capitalist economies ended. However, we are still in the age of stagnation. In 1980s and '90s, there was the attempt to privatize and/or deregulate various parts of the economy. Certainly the US economy improved the macroeconomic performance during '90s. Nevertheless it could be said as a dream that everything goes well if we emulate the economy that the general equilibrium theory presumes. As is often pointed out, we live in a financialized global economy and the instability of the economy has been strengthened. Now we must ask the fundamental question: "where can we find the origin of instability of the market?"

Marx stressed the antagonistic character inherent to a capitalist economy and developed a crisis theory based on fundamental characteristic of a capitalist economy. Facing the great depression in '30s, Keynes showed that a capitalist economy is not a self-regulating system and put emphasis on the importance of investment demand. Harrod keenly clarified the knife-edge character of investment in a capitalist economy. Nobuo Okishio (1927-2003) can be placed in this theoretical tradition. However his constructive theory of growth and accumulation, in particular the theory of cumulative process of disequilibrium has remained totally unknown to western readers, for it was mainly written in Japanese. ¹ No paper, which examines the Okishio's theory of growth and accumulation, has been appeared as far as I know. I believe that the globalization itself is also necessary in the heterodox economics camp to facilitate the discussion among a wide variety of economics.

Many people still imagine that the market mechanism can work smoothly. It cannot be true, I believe. If it could be true, why so many unemployed in the world cannot find jobs? I know someone insists they can get jobs if they accept the wage to go down. Then the equilibrium may recover. But in what sense can it mean to be an equilibrium? Can people enjoy a stable life in this equilibrium?

When we try to re-examine the problem of instability of a capitalist economy in our age, we must not presume that a capitalist economy is the same as the market economy in general. We must not forget the fact that a capitalist economy is a society that is organized by Capital. Therefore capital accumulation is the core of the working of this system. From this point of view, Okishio's theory might be the good starting point for us to analyze the contemporary capitalism even if it is totally financialized. As he applied a mathematical method to develop his theory, it means it could be easy to bridge his theory and a variety of heterodox macroeconomics in the West.

We can admit that the western academic circles have known the name of Okishio as a result of the upsurge in political economy in the 1970s.² In Japan he has been counted among one of the outstanding Marxist economists of a theoretical bent. For example, Shigeto Tsuru (1984) made the following remarks in his survey article:

It was before MORISHIMA attempted to solve some of the Marxian theorem mathematically that Nobuo Okishio, as a Marxist economist, applied mathematical techniques to the task of straightening out many of the problems on which Marx's exposition was not clear-cut or did not appear to be consistent. Actually,

¹ The collection of Okishio's papers written in English was published in one volume. See Kruger and Flaschel (1993). The editors of this book mention in their foreword that the papers in it "shows quite a different and — to the editors — in many respects new aspect of his creative power."
² Indeed Okishio visited Cambridge for two times. He wrote as follows: "In 1965 and 1976 I had the chance to study in Cambridge. M. Dobb, J. Robinson, N. Kaldor kindly induce me to write papers and many friends gave me intellectual stimulus." (Kruger and Flaschel (1993, p. x)) A short biography written by him is included in Arestis and Sawyer (2000), which allows us to know the avenue that he had explored personally.
he was a pioneer in this field in Japan, having started to publish his work in the middle of 1950's on such problems as the relation of value and price and the rate of profit.³

Okishio's works on the theory of value and price (known as 'Fundamental Marxian Theorem') and his discussion on the tendency of the rate of profit to fall (known as 'Okishio Theorem') have been made available in English.⁴ There have been many discussions on these topics in the native English-speaking world.⁵

The aim of this paper is to present an outline of his theory of accumulation and crisis and to examine its relation to the macroeconomics in the heterodox tradition, keeping in mind what is stated above.⁶ This paper is organized as follows. In section 2 Okishio's approach to economics will be reviewed briefly, which might be helpful for understanding the background of his economics. In section 3 Okishio's theory of accumulation and growth will be introduced, in which the analysis of the cumulative process of disequilibrium is important. Section 4 aims to compare the analytics of Okishio's theory with a neo-classical model of growth and some traditions in heterodox macroeconomics. Section 5 examines the critique of Okishio from the point of Uno-ist view.⁷ Section 6 concludes the essay.

2. Problems and Methods of Okishio's Economics

2.1 Problems
Okishio asserts that we have to tackle the following seven problems when we analyze the specific socioeconomic society, for example, a capitalist economy.⁸

#1. "What is the necessary level of human ability to control nature for the society in question to become

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³ Tsuru (1984) p.305
⁴ The 'Okishio Theorem' asserts that: If capitalists introduce a new technique when and only when it is cost-reducing, then the equilibrium rate of profit will rise, given the real wage rate. See Okishio (1961), J.E.Roemer (1981) chapter 3, 4 and 5 for instance. Okishio (2001) reexamined his own 'Okishio Theorem' that was based on the two questions: (1) the constancy of real wage rate and (2) the establishment of new production prices with positive profit. Removing these assumptions, he concludes "when the real wage rate changes owing to capitalists' competition, competition among capitalists tends to bring about a zero profit situation if there is no technical change during this process."
⁵ Makoto Itoh (1981) also surveyed "the development of Marxian economics in Japan." From his Uno-ist point of view, he argues somewhat critically that:

"Nobuo Okishio also employs mathematical methods in his presentation of the quantitative relation between the rate of surplus value and the rate of profit: his discussion of the tendency of the rate of profit to fall also contains some interesting insights. However, Okishio's position seems occasionally to be of a neo-Richardian variant of Marxism although his subjective intent is certainly purely Marxist. This tendency may be the result of the fact he confines his contributions to quantitative, mathematical problems." (p.30)

⁶ The question: what does the heterodox tradition hold in common?, is not simply concerned with taxonomy. Apparently it is very important to reconstruct economic theory, but here I use this to represent such schools as post-Keynesian, Marxian, Structuralist economics and so on, which are critical to the orthodox, that is, the so-called neo-classical economics. See Lawson (2003), Lee (2001,2002).
⁷ In Japan two economics society is competing: Japan Society of Political Economy (JSPE) and Japanese Economic Society (JES). The member of JSPE do the research basically from Marxian point of view, however, there have been long dispute on how to construct the economics system between orthodox Marxian economist and Uno School. Makoto Ito who has published many works in English and is well known in the West. So it might be helpful for the reader who knows Ito's work to understand the critique of Okishio from Unoist standpoint. As for the Japanese reconstruction of political economy, see Albritton (1986).
⁸ This section is a brief summary of Okishio (1989).
established and work? At such a stage what kind of production relations, especially property relations of the means of production, is efficiently workable and necessary?"

#2. “What other kind of production relations are derived from the property relations of the means of production which characterize the society? How do people act in such production relations? Who grasp economic decision-making and how do they decide?”

#3. “What economic phenomenon appears as a result of people’s actions? When treating capitalistic society, we must explain exchange, money, wages, profit, accumulation, crisis, business cycles, unemployment, technological innovation and so on from the basic characteristics of a capitalistic society.”

#4. “How are economic phenomena, caused by people’s actions, mutually interrelated? The analysis of simultaneous and intertemporal relationships is important. And mathematical tools are helpful especially in this field.

#5. “How do the thus caused economic phenomena function to maintain and reproduce production relations which characterize the society? In other words, we need to reveal the built-in reproduction mechanism of the society. For example, in a capitalistic society even economic crisis, unemployment and low wages function to maintain capitalistic production relations.”

#6. “How do economic phenomena caused in the society change man’s ability to control nature? What is the mechanism, which increases the productive forces? What is the threshold value of the ability to control nature which leads to incompatibility with the society in question?”

#7. “How do the economic phenomena caused in the society mature the factors which sublate the production relationships in the society? For instance, a capitalistic society gives birth to the human capacity to control nature, which contradicts with capitalistic production relations, and yet on the other hand, creates people, which has the awareness and capability to produce new production relations.”

[Insert Figure 1 here]

Among the list above, the problem #1 concerns the basis of existence for a specific society. The problems from #2 to #5 constitute the mechanism of social reproduction and the last two (#6, #7) constitute the mechanism by which the social form is replaced with another one. In summary, he argues that we should analyze a specific society from multiple perspectives: one is a viewpoint of “existence and reproduction” and another one is “transition”.9

2.2 Methods
The way of thinking, which Okishio applied, could be said to be fundamentally Marxian; in a sense that he thought he could analyze a specific society from two basic relations; productive forces and production relations. However, he was not an ordinary Marxian economist. The origin of Okishio’s economics is not only from Marx but also from the works by Hicks, Keynes and Harrod. He learned from Hicks (Value and Capital, 1946) “the capacity to analyze the interdependence of economic phenomena and to utilize mathematical methods for economic analysis”, and was indebted to Keynes (The General Theory of Employment, Interest and Money, 1936) for realizing the idea of “the importance of investment decisions for macroeconomic behavior”. Besides Hicks and Keynes, he owed Harrod (Toward a Dynamic Economics,

9 Looking back over his research, Okishio (1993) classified his publications into the following four groups: (1) the instability of a capitalist economy, (2) the labour theory of value (“FMT”), (3) ownership and decision-making and (4) Technical change and a capitalist economy (“Okishio Theorem”).
1949) who taught him a very important thing: the contradictory aspect of a capitalist economy.

At first Okishio regarded Marx's economics (Das Kapital) as "old-fashioned and obsolete". However, he finally recognized "its logical foundation is very tough and the basic propositions are capable of being derived by mathematical reasoning" and could find the argument on the transition of the social system itself that cannot be found in the works by Hicks, Keynes and Harrod.

Accordingly, we can say that Okishio's attempt had been to re-construct Marx's economics using mathematical methods and at the same time he introduced some ideas from non-Marxian economics when he judged them useful for his analysis of a capitalist economy. His first paper on Marx's proposition demonstrated by using mathematics was published in 1954, which preceded to Morishima's Marx's Economics (1973) by about twenty years. Okishio was one of the pioneering mathematical Marxian economists in the world as Tsuru has pointed out.

3. Okishio's Theory of Accumulation

Okishio developed his theory of accumulation and crisis in his book 'Chikuseki-ron ('Theory of Accumulation') which was first published in 1967 and revised in 1976. The discussion will be based on the revised edition.

3.1 A Brief Sketch of Okishio's Theory of Accumulation

We will give a brief sketch of Chikuseki-ron here and in the subsequent sections we will examine the relations between Okishio's theory and other theories in detail.

In chapter 1, he spells out how peculiar capitalist society is. These forces of production must stay within a certain limit of productivity in order for the capitalist society to exist and work. Capitalist society is a class-society where the production is governed by commodity-economic principles. In this peculiar society it is not satisfactory to exploit workers in the productive process in order to continue to maintain the exploitation. It is necessary to sell the product which workers produce, or in other words to realize the value of the commodity in the markets. And the most important item of demand for commodities in the capitalist economy is the investment demand by the capitalists. Accordingly the accumulation demand is the crucial element, which determines the level of the realized profits, the rate of profit and the real wage rate. The movements of the rate of profit and the real wage rate depend on how investment demand changes. The rate

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10 This section is mainly based on Okishio (1976), which is not available in English unfortunately. But the main points of his discussion are explained in the papers collected in Kruger and Flaschel (1993), especially part V (''Trade Cycle''). The book on trade cycle edited by Okishio (1992), especially Chapter 1(''Capitalist Society and Crisis''), is helpful, too.

11 The content of his book is as follows:

Introduction
Chapter 1 The fundamental structure of the capitalist economy
1. main features of the capitalist economy 2. the existence condition of profits
Chapter 2 Extended reproduction in the capitalist economy
1. A reproduction in general 2. A critical survey of theories of accumulation
Chapter 3 Capitalist accumulation and crisis
1. A cumulative process of the disequilibrium 2. A critical survey of theories of crisis
3. Crisis: the upper turning point 4. Upturn: the lower turning point
Chapter 4 Secular trend of the capitalist accumulation

Mathematical Appendix
of profit and the real wage rate display cyclical behavior since investment demand is volatile in the capitalist economy. The reason why this is so is explained in detail in chapter 3.

For any society reproduction is a question of vital importance for its continued existence. In chapter 2, Okishio explains the general conditions for reproduction regardless of the particular society and specifies the peculiar manner in which those conditions are satisfied in a capitalist economy. Those conditions for reproduction are always breached in the short run. Disequilibrium and instability are most probable since there is no conscious social control. However, unless conditions for reproduction can be satisfied on average in the long run, the capitalist society cannot continue to exist. This contradiction is inherent to capitalism. How does the capitalist society resolve this in the real world? In order to answer the question, Okishio begins with the most abstract level of theory and moves to the more concrete. First, in section 4 of chapter 2, he explores the properties of the growth path, which continues through time to satisfy the three conditions: (i) a product markets equilibrium, (ii) a normal utilization of capital and (iii) a constant rate of unemployment. This path is defined as the 'Equilibrium Accumulation trajectory'. It is interpreted as the growth path along which capitalist economy would move in the long run on average. Secondly he demonstrates, in chapter 3 that a dynamic process sets off which continually moves the economy away from the equilibrium once the economy deviates from equilibrium. 12 The disequilibrium (e.g. under utilization of capital, unemployment of labour and so on) spreads cumulatively in the capitalist economy. If, for any reason, the investment demand sets and is maintained above (below) the equilibrium level, it will increase (decrease) and will go on increasing (decreasing), and as a result, the production, the rate of profit, the real wage rate and the rate of exploitation rises (falls) cumulatively. ('The Cumulative process of disequilibrium')

'Violent equilibrating forces', which could reverse this upward (downward) cumulative process of disequilibrium, must be necessary in order for the capitalist society to reproduce itself. 13 This is why cyclical fluctuations are endogenous in the capitalist society, in other word, crisis and upturn are inevitable.

Through the cyclical movements, with recurrent unstable phases of expansion and stagnation, the capitalist economy expands and innovates the existing production techniques. How does the capitalist economy develop a tendency to move through the trade cycles? 14 The following phenomena, for example, will be observed: a deepening difficulty to find markets, the tendency towards a falling rate of profit and rising minimum size of a fund necessary in the basic sectors and so on. 15 Chapter 4 deals with the above questions.

As is easily noticed, the line of argument can be characterized as fundamentally Marxian. But his theoretical originality consists in the theory of accumulation, in particular, the mathematical analysis of the mechanism of reproduction in a capitalist economy, which will be presented in the following subsections.

3.2 A Harmonious Extended Reproduction Trajectory

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12 Heterodox economists often criticize the concept of equilibrium. However, Okishio does not accept this critique. He argues: "I think that it is not reasonable to refute the concept of equilibrium completely. Since capitalism has maintained itself for more than one hundred years, there must exist a mechanism for its reproduction. In order to analyze this mechanism, the concept of equilibrium is indispensable." (Arestis and Sawyer (2000, p.471))

13 cf. Capital III, p.249. "The crises are always but momentary and forcible solutions of the existing contradictions. They are violent eruptions which for a time restore the disturbed equilibrium."


15 "A certain stage of development come into conflict with the existing relations of production or — this merely express the same thing in legal terms — with the property relations within the framework of which they have operated hitherto. From forms of development of the productive forces these relations turn into their fetters. Then begins an era of social revolution. The Changes in the economic foundation lead sooner or later to the transformation of the whole immense superstructure."

For simplicity of the argument, we assume a one-sector economy with a fixed coefficient production technique. If we denote \( a \) and \( r \) the amount of commodity and labour directly needed to produce a unit of commodity respectively, we obtain the following:

1. \( K_i = aX_i \)

2. \( N_i = rX_i \)

We can interpret (1) as the condition of normal utilization of capital equipment. When there exists an amount of capital equipment \( K_i \) and we utilize it 'normally', we can produce \( \frac{K_i}{a} = X_i^* \). However, in reality the actual level of production \( X_i \) cannot be always equal to \( X_i^* \). So the ratio can be defined as the rate of utilization (\( \delta \)).

3. \( \delta = \frac{X_i}{X_i^*} \)

So if the rate of utilization is equal to unity, the above equation can be the same as (1). We assume that the workers consume all the wages, which is taken as constant here, and capitalists' divide their income in a constant proportion. In this case the conditions of product market equilibrium are formulated as:

4. \( X_i = aX_i + R_iN_i + I_i + C_i \)

5. \( R_i = R \) (const.)

6. \( I_i = \mu C_i \)

The capital accumulations is:

7. \( K_{i+1} = K_i + I_i \)

The system, which consists of six equations (equations (1), (2), (4)-(7)), suffices to determine the movement of six variables \( (X, K, N, R, I, C) \). The growth path, which is depicted by this system, satisfies the conditions of (i) equilibrium of product market, and (ii) normal utilization of capital through time. We call this growth path a **Harmonious Extended Reproduction Trajectory**.

Inserting (1), (2), (5) into (4) and considering the relation (6) and (7), the rate of capital accumulation can be obtained.

8. \( g_i = \frac{I_i}{K_i} = \frac{(1 - a - rR)\mu}{a(1 + \mu)} \)

If we denote the quantity of labour, which is necessary directly and indirectly to produce a unit of commodity, as \( t \), it is calculated as follows:

\[\text{Footnotes:}\]
\[16\] The following discussion is very limited in the sense that it abstracts the relationships of production sectors and technical change. But the main point of this section is to introduce the basic logic of Okishio's theory. The one sector model is satisfactory for this purpose. The model with technical change is examined in Appendix A.

\[17\] We assume the production period is unity, in other words, there is no fixed capital. So the expression of "the rate of utilization" might not have its original meaning. Here it means only the ratio of actual production to normal level of production.
\[ t = at + \tau \]

If the inequality: \( 1 - a > 0 \) (i.e. the condition of net production possibility) holds,

\[ t = \frac{\tau}{1 - a} \]

Here the surplus condition means that the inequality: \( 1 - R_t > 0 \) holds. This means the rate of exploitation is positive. Because the rate of exploitation is defined as

\[ e = \frac{1 - R_t}{R_t} \]

Then we can rewrite (8) as:

\[ g_t = \frac{I_t}{K_t} = (1 - a)(1 - R_t)\mu \]

If both the condition of net production possibility and the surplus condition are satisfied, \( g^* \) must be positive unless the investment demand is equal to zero. In sum the surplus condition means that the Fundamental Marxian theorem holds. As is easily demonstrated, the output, employment, investment and capitalist consumption grow at the same rate of \( g^* \).

If the actual economy were to move along the harmonious extended reproduction trajectory, it seems that there would be no problem. Because the capital equipment is utilized normally and then the commodity produced can be completely sold. If there is no technical change and capitalists' investment/consumption ratio is constant, \( g^* \) is determined by the real wage rate.

\[ (8') \quad g^* = \frac{(1 - a)(1 - R_t)\mu}{a(1 + \mu)} = g^*(R) \]

Apparently \( g^* \) would be higher if the real wage rate would be smaller. Even if \( g^* \) may be much high, there might be no inner contradiction from the point of the realization of the value of commodities. However, the demand for labour grows at the same rate \( (g^*) \) when the economy moves along the harmonious extended reproduction trajectory. The supply of labour is assumed to grow at the constant rate \( (n) \).

\[ (9) \quad L_t = L_0(1 + n)' \]

We have three cases of possible ways in which the economy will move.

**Case1:** \( g^* > n \)

Sooner or later the economy may be restricted by limited labour supply. Once the economy hits this ceiling, it will not be able to move along the harmonious extended reproduction trajectory.

**Case2:** \( n > g^* \)

As time goes on, unemployment will increase cumulatively, so the capitalistic relation of production will not be able to be maintained in the long run.

**Case3:** \( g^* = n \)
The rate of unemployment remains constant. The economy can move along the harmonious extended reproduction trajectory, which is defined as Equilibrium Accumulation Trajectory.

3.3 Equilibrium Accumulation Trajectory (EAT)
An Equilibrium Accumulation Trajectory is a growth path that satisfies three conditions: (i) an equilibrium of product market, (ii) a normal utilization of capital through time and (iii) a constant rate of unemployment, as already stated in the previous section. The actual accumulation path in a capitalist economy does not always move along EAT. EAT shows how the actual capital accumulation process can proceed on average in the long run while it exhibits the cyclical movement in the short run.

The growth rate of labour supply must be smaller than a specific value. It is necessary to satisfy the following relation for the existence of EAT.

\[ g^*(R) = \frac{(1-a)(1-Rt)\mu}{a(1+\mu)} = n \]

The maximum can be obtained when the real wage rate is zero and the capitalist investment/consumption ratio is infinity.

\[ g^*(0) = \frac{(1-a)\mu}{a(1+\mu)} \rightarrow g_{\text{max}} = \frac{1-a}{a} \text{ as } \mu \rightarrow \infty \]

Therefore, EAT cannot exist if the growth rate of labour supply exceeds \( g_{\text{max}} \), because the condition: \( g^* = n \) cannot hold. The rate of unemployment necessarily rises as time goes on. If the labour supply remains constant, the capitalist economy cannot realize extended reproduction in the long run unless labour saving technical change occurs.

3.4 The Cumulative Process of Disequilibrium
One of the most important characteristics of Okishio’s theory of accumulation is its emphasis on the distinction between those causes that trigger crisis and its necessity. His demonstration of the necessity of crisis consists of two parts. First, he shows that the disequilibrium, once it occurs, spreads cumulatively in the capitalist economy, and that is caused by the fundamental contradiction of the capitalist economy. Next, he argues that if this cumulative process of disequilibrium goes on unceasingly, the conditions for reproduction of the capitalist economy will be destroyed. Accordingly from these two assertions, we can say that a crisis is inevitable as long as we assume the continued existence of the capitalist economy.

Now we show the necessity of the cumulation of disequilibrium mathematically.

Product market equilibrium:

\[ X_t = aX_t + R_tN_t + I_t + C_t \]

Rate of profit:

\[ r_t = \frac{X_t - aX_t - R_tN_t}{K_t} \]

Production technique:

\[ X_t = \delta_t \frac{K_t}{a} \]
(15) \( N_t = rX_t \)

Rate of accumulation:

(16) \( g_t = \frac{I_t}{K_t} \)

Investment function:

(17) \( g_{t+1} = g_t + \beta(\delta_t - 1) \quad \beta = \text{const.} > 0 \)

Degree of utilization:

(18) \( \delta_t = \delta(r_t) \quad \delta'(r) > 1 \quad \delta(r^*) = 1 \)

Capital Accumulation:

(19) \( K_{t+1} = K_t + I_t \)

Capitalist Consumption:

(20) \( C_t = \frac{1}{\mu}I_t \quad \mu = \text{const.} > 0 \)

The model that consists of (12) – (20) suffices to determine the actual growth path. Equation (12) states that the level of production is determined so as to satisfy the demand if the demand is below the production capacity. Equation (13) defines the rate of profit. Equations (14) and (15) state that the production technique is the so-called Leontief-type. Equation (16) defines the rate of accumulation. Equation (17) is the investment function, which is often called ‘Harrod=Okishio type’. This investment function shows that “capitalists determine the rate of accumulation at the same rate as the previous period if the capital equipment is normally utilized, at a greater rate if over utilized and at a smaller rate if underutilized.” \(^{18}\) The degree of utilization function (18) shows that capitalists require a certain level of the rate of profit in order to carry out the production. That is, if the actual level of profit is above (equal or below) the required level \( r^* \), the degree of the utilization is above (equal or below) the normal level. However, there would be a physical upper limit \( (\bar{\delta}) \) for utilization.

When the economy moves along the path that satisfies both a normal utilization of capital and the equilibrium of commodity market (i.e. Harmonious Extended Reproduction Trajectory), the rate of profit must be the required level.

(21) \( \delta(r_t) = 1 \text{ i.e. } r_t = r^* \)

From (12), (13), (16) and (20), we get

(22) \( g_t = \frac{\mu}{1 + \mu} r_t \)

So the rate of accumulation must be constant \( (\frac{\mu}{1 + \mu} r^* = g^*) \). On the other hand, considering Equations

\(^{18}\) Okishio (1967) p.21
(13) – (20), (12) can be rewritten as:

\[
\frac{1-a-\tau R}{a} \delta_t = \frac{1+\mu}{\mu} g_t.
\]

Therefore the real wage rate must be equal to \( R^* \) which satisfies the following:

\[
\frac{1-a-\tau R^*}{a} = \frac{1+\mu}{\mu} g^*.
\]

From (17), (18) and (22), we can get the difference equation as:

\[
g_{t+1} = g_t + \beta(\delta(\frac{1+\mu}{\mu} g_t) - 1)
\]

Equation (25) implies that if the initial value of \( \delta \) is less than unity, that is the initial value of the rate of accumulation is smaller than the equilibrium level, then \( g_t \) becomes smaller as time goes by and vice versa. This can be shown as Figure 2.

[Insert Figure 2 here]

The slope of the curve A is greater than unity, because

\[
\frac{dg_{t+1}}{dg_t} = 1 + \beta \delta \frac{1+\mu}{\mu} > 1 \quad : \quad (17), (18)
\]

We know, from (22), that when the rate of accumulation diverges from the equilibrium, so does the rate of profit. The reason why the actual growth path is unstable is that capitalists accumulate according to the 'Harrod=Okishio type' investment function. That capitalists behave as depicted as (17) is rooted in the peculiar manner of capitalist production.

The origin of this cumulative movement must be explained. It can be summarized graphically as Figure 3.\textsuperscript{19}

[Insert Figure 3 here]

- If a capitalist economy were not a class society, in which products are produced for exchange purpose, the products cannot be commodities, and no problem of realization of commodities can exist. Therefore no disequilibrium of commodity realization exists.
- If the workers were not exploited and their consumption demand is limited, no difficulty of commodity realization exists and the investment demand by capitalists cannot play a critical role in the determination of the aggregate demand.
- If the decisions on production were not determined in pursuit of private profit on limited local information, or there exist anarchy of production, the investment demand can move so as to restore the equilibrium even if the disequilibrium were to emerge.

\textsuperscript{19} Okishio (1976) pp.199-200.
In a word, the cumulative movement of disequilibrium originates from the basic characteristics of a capitalist economy.

3.5 The Behavior of the real wage rate

Here we examine the behavior of real wage rate in the cumulative process in detail. Rearranging (23), it follows that:

\[(27) \quad R_t = \frac{a}{\tau} \cdot \frac{1 + \mu}{\mu} \cdot \frac{g_t}{\delta_t} + \frac{1 - a}{\tau}\]

From (21) and (22), we obtain the following:

\[(28) \quad \frac{dR_t}{dg_t} = -\frac{a}{\tau} \cdot \left(\frac{1 + \mu}{\mu}\right)^2 \cdot \left(\frac{\mu}{1 + \mu} \cdot \frac{g_t}{\delta_t} \cdot \frac{d\delta_t}{dg_t}\right)\]

The sign of \(\frac{dR_t}{dg_t}\) is positive if the elasticity of \(\delta\) of the rate of accumulation is larger than \(\frac{\mu}{1 + \mu}\) (or if the elasticity of \(\delta\) of the rate of profit is larger than \(\frac{\mu}{1 + \mu}\)). Clearly the rate of utilization cannot rise unceasingly and so we denote the physical upper limit of \(\delta\) as \(\bar{\delta}\). As the upward cumulative process continues, \(\delta\) necessarily encounters \(\bar{\delta}\). In these situations, as the investment function (17) cannot work, instead, we postulate the following:

\[(29) \quad g_{t+1} = g_t + \gamma(r_t - r^*)\]

In this case we obtain,

\[(30) \quad \frac{dg_{t+1}}{dg_t} = 1 + \gamma \cdot \frac{1 + \mu}{\mu} > 1\]

\[(31) \quad R_t = -\frac{a}{\tau} \cdot \frac{1 + \mu}{\mu} \cdot \frac{g_t}{\delta} + \frac{1 - a}{\tau}\]

Since the degree of utilization of capital is equal to the upper limit, the real wage rate necessarily falls. Therefore it is shown that the real wage rate should fall in the end if the upward cumulative process of disequilibrium goes on unceasingly. Similarly, the real wage rate should rise in the downward cumulative process of disequilibrium.

As is well known the most important condition for a capitalist economy to reproduce is the existence of workers who sell their labour power. This is guaranteed if the real wage rate is limited within a certain range. It must not become below a minimum level \((R_{\text{min}})\) that is determined by physical, cultural and social considerations. At the same time it must not exceed a maximum level \((R_{\text{max}})\) that allow workers to buy the means of production and that imply capitalists do not exploit workers. The latter means that the surplus condition \((1 - R_t > 0)\) must be satisfied. That is, the real wage rate must be a level such as:
(32) \( R_{\text{min}} \leq R \leq R_{\text{max}} < \frac{1}{t} \)

If this upward cumulative process of disequilibrium continues unceasingly, the real wage would fall below the minimum level, a full-employment ceiling would be binding and so on. Then, for example, the time when labour power cannot be reproduced necessarily comes. Hence, in order the capitalist society itself reproduce, some cause or other must reverse this upward cumulative process of disequilibrium.

In summary, two propositions have been demonstrated so far. These are,

- Proposition (a): A cumulative process of disequilibrium inevitably proceeds in a capitalist economy.
- Proposition (b): The reproduction of a capitalist economy becomes impossible if the cumulative process proceeds unceasingly.

Accordingly, we can say that a capitalist economy is obliged to have a trade cycle.

3.6 Causes of crisis and upturn

In this section, we discuss the causes of crisis and upturn. The characteristic of Okishio’s theory of accumulation is that it separates the concrete moments of crisis from its necessity. He asserts that the moments, which reverse the cumulative process, cannot be predetermined \textit{a priori}.

"It is not correct to pick up one specific moment which reverses the cumulative process and then generalize that the cumulative process is necessarily reversed by this moment. We cannot deduce a unique moment of reversal merely from the fundamental characteristics of a capitalist economy." \(^{20}\)

Then Okishio (1976) lists five moments, which are most probable, triggering the crisis. These are as follows:

(i) Over-production in the consumption goods sector
(ii) Under-production in the capitalist goods sector
(iii) Labour shortage; full-employment ceiling
(iv) Credit restriction
(v) Lower limit of the real wage

Let us discuss the causal relationship, which triggers the crisis. By way of example, we examine the ‘labour shortage’ case. When capitalists cannot employ workers, as they want, this constraint would bring about any shift in their plan to extend production in the subsequent period. This would lead to a fall of investment demand and trigger a downward movement. Facing the shortage of labour, capitalists will be engaged in competition for labour. In this labour poaching competition, capitalists in the consumption goods sector will be likely to be defeated. This causes an idle capacity of equipment in the consumption sector and a fall of investment demand. And in response to this fall in the consumption goods sector, investment demand also would decline in the capital goods sector. This triggers the crisis.

The necessity of the upturn (the upper turning point) can be demonstrated in the same way of the case of crisis. As the causes of upturn, Okishio (1976) listed the following five moments:

\(^{20}\) Okishio (1992) p.11.
(i) Prosperity of the consumption goods sector  
(ii) Dissolution of the bottleneck  
(iii) Investment demand to introduce a new production technique  
(iv) Capitalists' consumption demand  
(v) Replacement demand of equipment

From above discussion, we can say that cyclical movements are endogenous in the capitalist economy and this is rooted in the peculiarity of the capitalist manner of production and accumulation.

3.7 Trade cycle
Now we study the behavior of the major economic variables in a cycle. The movement of the rate of accumulation, the rate of profit, the rate of exploitation, the rate of unemployment and the real wage rate are shown in Figure 4. All variables exhibit cyclical movements. Any point of time in the course of trade cycle cannot satisfy the conditions for harmonious reproduction. The economy is always in the upward or downward cumulative process of disequilibrium. The fact that social reproduction in a capitalist economy is realized through the cyclical movements means some portion of the workers is always unemployed on average in the long run.

[Insert Figure 4 here]

4. Analytics of Okishio’s Model

A variety of perspective has been explored in the heterodox macroeconomics, for example, post-Keynesian, Marxian, US Radical Economics, and Structuralist Macroeconomics. In this section, first, the causal economic process of Okishio’s model will be compared with the ‘the Classical Marxist Macroeconomic Model’, which Palley (1999) presented. Then an analytic contour that differentiates them will be examined.

4.1 Causal economic process
The causal relationships among variables can be summarized as Figure 5. There are many noticeable features in this macroeconomic process.

- The direction of causality runs from the good market to labour market. The financial market does not play an active role in this process.
- Conditions in the goods market determine the real wage rate and employment. The labour market itself is behind the scenes in this model. Workers' behavior has no influence on the determination of the real wage and employment.
- Investment demand is decisive in the determination of production.
- The major determinant is the profit rate, so it is central to the account of the behavior of the capitalist economy.

[Insert Figure 5 here]

According to Palley (1999), the classical Marxist macroeconomic process can be represented as Figure 6 in a
"schematic" way. From the view of causal relationships, this process by comparison to Okishio's model above is characterized as follows.  

- "Causation flows 'out' of the labour market and the supply side of the economy into the rest of the economy."
- "Profit rates determine the interest rate. Thus, finance is very much super-structural."
- "Considerations of aggregate demand are absent."

[Insert Figure 6 here]

If we accept the schematization by Palley, Okishio is a "classical Marxist" in the sense that he emphasizes the centrality of the profit rate for the understanding of a capitalist economy and its relationship to the class relation. But he is rather Keynesian in his emphasis on the importance of investment demand.

4.2 Differences in how to describe a capitalist economy

When we put a variety of models of accumulation and growth to test, we have to have some yardstick to judge. Okishio's theory is characterized by contrast with a neo-classical growth model and a recent neo-Kaldorian model, in particular. In doing so, we focus on the characteristic of the growth path and its assumed institutional configurations.

First, we formulate the standard Solow-type neo-classical model as follows.

\[ (33) \text{Equilibrium of Goods market: } sY = I \]
\[ (34) \text{Production Function: } Y = F(K, e^{aN}) \]
\[ (35) \text{Rate of Profit: } r = \frac{Y - RN}{K} \]
\[ (36) \text{Technical Choice: } R = F_N \]
\[ (37) \text{Supply of Labour: } \hat{N} = n \]
\[ (38) \text{Capital Accumulation: } \dot{K} = I \]

The assertion of a neo-classical model is that there exists the equilibrium growth path that satisfies three conditions (i.e. (i) equilibrium of goods market, (ii) normal utilization of capital and (iii) full employment of labour) and an economy converges to this path. If we assume Harrod-neutral technical change, the growth rate of output, capital and so on is equal to the growth rate of labour supply plus the rate of technical progress. At first sight this result seems to be the same as the Equilibrium Accumulation Trajectory. However we must

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21 As Palley (1999) does not present a mathematical model, it cannot compare rigorously. However, the main point is to highlight the character of Okishio's model, so this presentation is sufficient for this aim.
remind of the condition of a constant rate of unemployment in the latter. The EAT is the growth path on which the actual economy realizes on average in the long run. So if no reserve army is available on the EAT, the economy cannot experience the upward move. Moreover, the growth path in a neo-classical model is considered as the actual on, on the contrary the EAT is not the actual path in Okishio’s model. Though a neo-classical model concludes the stability of the equilibrium growth, this stability is completely different from the so-called stability in a sense of Harrod. In the latter the problem is whether or not the disequilibrium such as excess demand and/or excess supply can be diminish. But the path that a neo-classical model describes always satisfies the equilibrium conditions. The problem in a neo-classical model is whether or not the growth path will generate some contradiction that prevents the economy to move on.

The level of production is determined so as to satisfy both the normal utilization of capital and full employment of labor. And then the investment is adjusted to achieve the equilibrium of product market (Say’s law). So we can find no independent behavior of capitalists and no capitalists who exercise power. In equilibrium the product and capital grows at the same rate; the rate of profit remains to be constant; and the real wage rate increases at the rate of technical progress, which is a very harmonious state.

In sum, we have to admit that a neo-classical growth model is a theory of a capitalist economy where capitalists are absent.

Recently Setterfield and Cornwall (2002) extend the traditional Kaldorian model of cumulative causation by introducing institutional considerations. However, they use a basic model with different parameter set to analyze the different regime, that is, the ‘Golden Age (GA)’ and the ‘Age of Decline (AoD)’. Essentially the model they apply is the traditional one. So it suffices to examine the difference between a neo-classical model and traditional Kaldorian model. Their ‘Macroeconomic Regime (MR)’ consists of Productivity Regime (PR), Demand Regime (DR) and Institutional Regime (IR).

\[(39)\] PR: \( q = r + \alpha y \)

\[(40)\] DR: \( y = \Omega + \lambda \omega x \beta q \quad \Omega = \lambda (\omega x a - \omega x \beta r + \omega x (\gamma - \alpha \omega)y_0) \)

where \( q \) denotes productivity growth and \( y \) is the rate of growth of output. Using this framework, they tried to clarify the cause of breakdown of the Golden age and the viability of neo-liberal regime. But the point here is to understand the characteristics compared with a neo-classical. The first we have to notice is that their approach rests on the notion that “capitalism is not self-regulating.” And in addition, the following two points are identified as “fundamentals of a capitalist economy.”

- “the actual rate of growth is demand determined.”
- “the division of labour depends on the extent of the market.”

Neo-Kaldorian model differentiates from a neo-classical in this sense, that is, they put stress on the unstable nature of a capitalist economy and its demand-led nature. Nevertheless the equilibrium that is defined by (39) and (40) is assumed to be stable. But this stability can be realized under some specific institutional setting.

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so it is not mechanical stability in a neo-classical model.

As for the investment behavior that we identify the most important in a capitalist economy, we cannot find any explicit investment function in this model. They stress the priority of demand in determination of production, though we have to say they fail to understand the “fundamentals”. Goodwin (1967) proposed a Marxian type model, which stresses on class struggle and on the importance of the reserve army of labour, but Say’s law and the normal utilization of capital are assumed. Accordingly his model succeeded in describing one important aspect of a capitalist economy (class society), but it failed to incorporate the independent role of investment behavior. However, Skott (1989a) integrates the Keynesian idea on effective demand with a Marxian stress on class struggle. 23 Though Skott does not examine the fundamental characteristics of a capitalist economy as Okishio did, his model is place to the similar line of Okishio’s argument.

4.3 Investment, Uncertainty and Finance

Investment demand is the major determinant of the aggregate demand, so the production and employment. Financial factors are not included in the formulation of the Harrod-Okishio type of investment function. On the other hand, the “credit restriction” is counted as one of the moments that triggers crisis. We have to examine the capitalist investment behavior in detail.

First of all we have to distinguish (a) what sort of investment capitalists do and (b) how capitalist obtain the fund for investment. 24 These are completely different problems. As for problem (a) we assume the following Harrod-Okishio type of investment function. The rationale of this function can be explained as follows.

- The major determinant of investment demand is the relation between production capacity and actual production (the aggregate demand), that is, the rate of utilization.
- The purpose of investment in the present period is to increase the production capacity in the next period. But the capitalist cannot know the level of demand for her product in the next period or even that in the present period. She has to determine these based on her private expectation.
- For a first approximation, we assume static expectations, that is, the rate of utilization in the present period is equal to that in the previous period. Therefore the production capacity in the next period is determined by the information about two periods ago. This is one reason that causes the cumulative movement of disequilibrium.
- What is determined by the rate of utilization is not the absolute level of investment but the ratio of investment to capital stock (i.e. the rate of accumulation). It might be rational to think that even if the rate of utilization is the same, investment must be higher in case where there is much capital equipment.
- As for the functional relationship between the rate of accumulation and the rate of utilization, the capitalist raises (maintains, lowers) the rate of accumulation when the rate of utilization exceeds (is equal to, falls short of) unity.
- The purpose of production is the pursuit of profit. So we must recognize that a capitalist raise the

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23 see also Skott (1989b).
24 The following discussion on the relationship between the investment and a loanable fund depends mainly on Okishio (1976, pp.187-193).
rate of utilization only if she can earn more profit. Thus we can assume that the rate of utilization is an increasing function of the realized rate of profit as a first approximation.

Accordingly we can formulate the Harrod-Okishio type of investment function mathematically as:

\[
\frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + F(\delta^*) \quad F(1) = 0 \quad F'(0) > 0
\]

(41) \( \delta^* = \delta_{t-1} \)

\[
\delta_{t-1} = \delta(r_{t-1}) \quad \delta' > 0
\]

If we denote the rate of accumulation as \( g \), the investment function can be formulated as:

(42) \( g_t = g_{t-1} + \beta(\delta(r_{t-1}) - 1) \quad \beta > 0 \)

This is what we assumed in the previous section. The investment function of Keynes is often assumed as:

(43) \( I_t = f(i_t) \quad f' < 0 \)

Certainly the investment depends on the level of interest rate, but the shape and the location of the function \( f(\cdot) \) has the vital importance, that is, the state of expectation, or "animal spirits". The uncertainty accompanying investment behavior has an important role in both formulations. Here again Okishio is in a tradition of Keynes as he himself admitted.

Next, we examine problem (b). Apparently the following constraint must hold.

(44) \( F_t \geq I_t p_t \)

\( F_t \): Maximum loanable fund for capitalist of this period,

\( I_t \): Investment demand, \( p_t \): the price of capital goods

If the investment demand determined by (44) exceeds \( F_t \), it must be lowered to satisfy this constraint. But the reverse cannot hold. Even if the maximum is large, the investment is determined by (44). The level of investment is irrelevant to \( F_t \). \( F_t \) is not limited by the money that a capitalist holds. Because she need not pay the cost of investment by money holdings, she can buy the capital equipment on credit. So the maximum credit limits the value of \( F_t \). Capitalists can take a loan from a bank and in addition they can make a mutual-credit. As long as the products can be sold smoothly, capitalists pay off the loan and continue to accumulate on credit. Thus the limit of the mutual-credit depends on whether the value of commodities can be realized or not. This depends again on the aggregate demand, and therefore in the end, upon investment demand. In summary, the limit of credit depends on investment demand itself. Here again, the investment demand is an independent variable. The maximum loanable fund for capitalists increases when the economy moves along the cumulative process of disequilibrium upward. The crisis stops the increase of loanable funds, and then triggers a collapse of credit, which is not the cause of the crisis but the result.
Dymski (1990) proposed a schema that aims to categorize money and credit theory in radical political economics. His criterion consists of three dimensions: the treatment of time, uncertainty and information. In the world of the Walrasian general equilibrium model that satisfies the assumptions of timelessness, perfect information and certainty, money and credit play no important role. He classified the works of Marglin, Roemer and Sraffa as the Marxian model incorporating the assumptions above. That is to say, in these models, all crises are always "real". Apparently we acknowledge that Okishio’s model does not fall in the same category as Marglin and so on. The importance of uncertainty in decision-making of investment is emphasized in the formulation of Okishio=Harrod investment function. But Okishio’s stance is different from the approach that admits the possibility that "financial relations may undermine accumulation"; that is the theory based on the Minsky’s notion of "financial fragility". Though Okishio does not deny the function of credit in the process of crisis, the point is whether the money and credit relation are relatively autonomous.

The main aim of Okishio’s model is to dissect the working of a capitalist economy at a highly abstract level. Contemporary crises, for example that of Japanese economy in the ’90s, can be characterized as financially driven in form. An abstract model alone cannot demonstrate everything. The task that remains is to fill the gap between the abstract model and its reality.

5. Uno-ist’s Critique of Okishio’s Theory

It is well known that Marx could not complete his own crisis theory fully during his life and he left different types of theory. According to Itoh (1981), they “can be divided into two major types.” “One is based on the excess commodity theory and the other on the excess capital theory of crisis.” Sub-branches of each type and the representative theorists of each branch in the history of economic thought are as follows:

A. The excess commodity theory
   1) The disequilibrium variant
      Tugan-Baranowsky, R. Hilferding
   2) The under-consumptionist variant
      K. Kautsky, R. Luxemburg, N. Bukharin, P. Sweezy

B. The excess capital theory
   1) The labour power shortage theory
      Ottor Bauer, Kozo Uno
   2) The rising composition of capital theory
      E. Preiser, H. Grossman

It would be necessary to examine fully what characteristics Okishio’s theory has, compared with other theories of crisis in Marxian camp. Here, I would like to confine myself to making a few remarks. I only discuss the difference between Okishio’s theory and the above B-1) type of theory, in particular Uno-ist theory of crisis.

From his viewpoint of the labour shortage type of crisis theory, Itoh regards Okishio’s theory of crisis as

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26 M. Itoh (1981) p.120
an attempt to reconstruct Marx’s excess commodity theory of the under-consumptionist type within the framework of Keynesian dynamics. Itoh criticizes Okishio on the basis that his theory suffers some drawbacks theoretically and in its reality. The points, which are thought to be doubtful, are as follows:

- Investment demand is treated as a completely independent variable, though it has been considered in the orthodox Marxian economics that a portion of the surplus value is invested or accumulated.
- The misery of the working class is supposed to increase absolutely at the last stage of the boom.

With regard to the first point, I think that Itoh’s critique may not be appropriate. First, some theorists have been thought that the fund for accumulation is supplied only by realized profits. But this is not true. For, before capitalists make a profit, they must accumulate in expectation of being able to sell their commodities. Secondly, though it may be thought that the accumulation demand is limited by the availability of loanable fund, the upper limit on credit is not restricted by the loanable ability of commercial bank but by trade creditability as discussed in the previous section. And the limit of this trade credit depends on whether the product can be sold smoothly or the value of the commodities could be realized easily. In other word, the limit of credit is determined by the aggregate demand. Here, the accumulation demand is an independent variable. In short, when the economy deviates from equilibrium in the direction of increasing excess demand in the product markets, the upper limit of credit also increases.

Next we examine the second point. Theories are divided conclusively on this point, that is, whether the real wage rate rises or not in the phase of prosperity. According to the labour power shortage theory, the composition of capital remaining the same is regarded as a main feature of smooth expansion and so the necessary accumulation necessarily accompanies the increased demand for labour power.

“The change in capital accumulation towards a higher organic composition of capital occurs when such a widening expansion of social productive power encounters limit in the size of the working population.”

“When the accumulation capital turns into excess accumulation, the rise in wages pulls down the rate of profit.”

We must interpret “the rising wages” in the above quotation as meaning ‘the rise in the real wage rate’. But the real wage rate is clearly equal to (money wage rate)/(price of consumption goods). When unemployment declines in the phase of prosperity, the money wage rate would rise but the real wage rate would not necessarily rises. If we can demonstrate that the growth rate of money wage rate is always larger than that of the price in the boom, we can say that real wage rate always rises. However, this is not done.

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27 Okishio (1976, p.208) himself states, “we must regard the under-consumptionist type as the theory, which does not demonstrate the necessity of the crisis but proves one possible cause of crisis.” He also states “restricted consumption of the working class is the indispensable condition that causes crisis.” (p.210) And he quotes from Capital “The ultimate reason for all real crises always remains the poverty and the restricted consumption of the masses as opposed to the drive of capitalist production to develop the productive forces as though only the absolute consuming power of society constituted their limit.” (Capital vol.3, chapter 30 p.484)


29 To put it mathematically, the rate of accumulation is the independent, not the dependent, variable; the rate of wages, the dependent, not the independent, variable.” The question is how we interpret the meaning of independent variable.

6. Concluding Remarks

The gist of Okishio’s theory of accumulation is to clarify the necessity of cyclical movement in a capitalist economy based on his Harrod-Okishio type of investment function. The realized rate of profit depends on the rate of accumulation, which is based upon decisions, which are private and dispersed. As a result, it generates a cyclical movement of basic economic variables. It can be said that his work is greatly indebted to Marx, Keynes and Harrod. Marx’s Das Kapital was published in 1867, and Keynes’s the General Theory in 1936. It is often argued that these works are too old-fashioned and obsolete, and that the same is also true of Okishio’s work. Their analysis cannot contribute to the analysis of the so-called financialized global economy in which we live now.  

We can summarize the main points of the perspective of Okishio.

- Centrality of Conflict
- Capital Accumulation as Demand as an Independent variable
- Technical Change as a Constraint upon Capitalistic Development

As for the first point, everybody knows there still exists a variety of conflicts in the world. The world is completely different from the picture that the neo-classical economics shows. Conflict and/or power issues are absent in orthodox economics. However it is indispensable for the understanding the contemporary capitalism even if it is globalised.

Secondly, the accumulation model examined so far abstracts the public sector and foreign sector. When we analyze the economy at a more concrete level, we have to take these factors into consideration. The idea that a capital accumulation as an independent variable is critical can be applied to the analysis of contemporary capitalism. The aggregate demand determines the realized rate of profits. Aggregate demand consists of replacement demand, workers’ consumption demand, capitalists’ consumption demand, investment demand, government expenditure and export. As is shown in Appendix C, the realized rate of profit after tax is determined by such factors as the rate of accumulation, the rate of government deficit spending, the rate of trade surplus, capitalists’ consumption and workers’ consumption.

\[
(45) \quad S_n \frac{\Pi - T_n}{K} + S_w \frac{W - T_w}{K} = \frac{I}{K} + \frac{G - T}{K} + \frac{E - F}{K}
\]

The rate of accumulation is the most important of these. If capitalists cannot maintain a high level of accumulation rate, the realized rate of profit must fall, which induces the rate of accumulation to fall and then the rate of profit will fall and so on; the downward cumulative process proceeds. From the above relation, we can understand how to raise the rate of profit after tax in face of a stagnant accumulation by:

- Increasing the rate of government deficits spending
- Increasing the rate of trade surplus

\[31\] For an analysis of the so-called ‘finance-led growth regime’ from a Regulationist approach, see Boyer (2000).
• Decreasing the workers’ income distribution
• Decreasing the propensity to save

The first two devices can be seen as a strategy of traditional Keynesianism. The mixture of the second and third is what recent Neo-liberalism pursues. It is apparent that these strategies are not effective in the long run. Because, the government deficit cannot be increased, and all the economies in the world cannot increase their trade surpluses at the same time. Such an expansionist policy will be constrained by environmental issues. Greater inequality in income distribution will heighten the social instability. The policy presumes that socio-economic institutions are capitalistic, and given. But we can discard this presumption and by doing so, we can pursue an alternative. The perspective that centers on accumulation provides us with a basis for evaluating current policies and the alternative.

We argue that one of the existence conditions for the equilibrium accumulation trajectory is that the type of technical change must be such that it keeps the organic composition of capital constant. We know the relation concerning the rate of profit always holds:

\[
(46) \text{Rate of profit} \quad r = \frac{M}{C + V} < \frac{V + M}{C} = \frac{N}{C}
\]

The upper limit on the rate of profit is the ratio of (living labour) to (dead labour) or the organic composition of production. If we rewrite this by our notation, it is expressed as:

\[
(47) \quad \frac{N}{C} = \frac{\tau X}{aXt} = \frac{\tau}{at} = \frac{1 - a}{a}
\]

That is to say, the rate of profit necessarily declines if the coefficient \((a)\) becomes smaller, which cannot be solved by a Keynesian policy. It is apparent this does not belong to a kind of realization problem.

The applicability of a specific theory should not be judged by its superficial resemblance to realities. We have to have theoretical apparatus that help us to understand the mechanism underneath 'realities'. In this sense the theory of growth and accumulation in the tradition of heterodox macroeconomics is still effective.

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Appendix A:

Technical Change and the Equilibrium Accumulation Trajectory in a One-sector Economy

1. Labour-saving technical change

Here we introduce the labour-saving technical change. The rate of exploitation is assumed to be constant instead of the assumption of constant real wage rate.

First the harmonious extended reproduction trajectory is characterized by the following system.

(A1) \( K_t = aX_t \quad a: \text{const.} \)

(A2) \( N_t = \tau_tX_t \)

(A3) \( \tau_{t+1} = (1-\alpha)\tau_t \quad \alpha > 0: \text{const.} \)

(A4) \( X_t = aX_t + R_tN_t + I_t + C_t \)

(A5) \( e = \frac{1 - R_t}{R_t} \quad (: \text{const.}) \)

(A6) \( I_t = \mu C_t \)

(A7) \( K_{t+1} = K_t + I_t \)

(A8) \( t_t = at_t + \tau_t \)

The system, which consists of eight equations, suffices to determine the movement of eight variables \((X, K, N, R, I, C, t, \tau)\). The growth path, which is depicted by this system, satisfies the conditions of (i) equilibrium of product market, and (ii) normal utilization of capital through time.

Inserting (A1), (A2), (A3) into (A4) and considering the relation (6) and (7), the rate of capital accumulation can be obtained.

(A9) \( g_t = \frac{I_t}{K_t} = \frac{(1-a-\tau_tR_t)\mu}{a(1+\mu)} \)

From (A8), we obtain,

(A10) \( R_t\tau_t = \frac{R_t\tau_t}{1-a} \)

As the rate of exploitation is assumed to be constant, the expression \( R_t\tau_t \) is also constant. Therefore the rate of accumulation in (A9) must be constant.

Then we can rewrite (A9) as:
\[
A11 \quad g_t = \frac{I_t}{K_t} = \frac{(1-a)(1-R_t,\mu)}{a(1+\mu)} = \frac{(1-a)e\mu}{a(1+e)(1+\mu)} (= g^*)
\]

If both the condition of net production possibility and the surplus condition are satisfied, \( g^* \) must be positive unless the investment demand is equal to zero. When the economy moves along this path, the production, capital equipment, investment, consumption grows at this rate. And the real wage rate grows at the same rate of labour productivity.

From (A2) and (A3), we find the growth rate of labour demand is equal to \( g^* - \alpha \).

The labour supply is assumed to grow at the constant rate \((n)\).

\[
A12 \quad L_t = L_0(1+n)^t
\]

Accordingly, in order to move along on the harmonious extended reproduction trajectory, the relation must be satisfied.

\[
A13 \quad g^* = n + \alpha
\]

This defines the Equilibrium Accumulation Trajectory. As the same as in the case that there is no technical change, the maximum rate of growth is:

\[
A14 \quad g^*(0) = \frac{(1-a)\mu}{a(1+\mu)} \rightarrow g_{\text{max}} = \frac{1-a}{a} \quad \text{as} \quad \mu \rightarrow \infty
\]

That is to say, the growth rate of labour supply must be lower than \( \frac{1-a}{a} - \alpha \) for the existence of equilibrium accumulation trajectory.

2. The type of technical change and equilibrium accumulation trajectory

We assume a pure labour-saving technical progress in section 1. Marx described the technical change by the organic composition of capital \((C/V)\), which is expressed as following by our notation.

\[
A15 \quad \frac{C_t}{V_t} = \frac{aX_t\tau_t}{R_t\tau_tX_t} = \frac{a}{R_t\tau_t}
\]

As is shown \( R_t\tau_t \) is constant under the assumption of constant rate of exploitation, the organic composition of capital is constant. How is the conclusion above altered if we assume a general type of technical change? Here the capital-saving technical change is also assumed.

\[
A16 \quad a_{t+1} = (1-\beta)a_t, \quad \beta > 0: \text{const}.
\]

Under these circumstances, the rate of accumulation is expressed as:

\[
A17 \quad g_t = \frac{I_t}{K_t} = \frac{(1-a-\tau_tR_t)\mu}{a(1+\mu)}
\]
If we maintain the assumption of constant rate of exploitation, we find the rate of accumulation is a decreasing function of \( a_i \). This means the rate of accumulation continues to rise so that the economy can move along the harmonious extended reproduction trajectory. The growth rate of labour demand must accelerate. On the other hand the growth rate of labour supply is constant by assumption. Then the rate of unemployment cannot remain constant. Accordingly no equilibrium accumulation trajectory can exist in this case.

This argument implies the type of technical change plays a decisive role in the reproduction of a capitalist economy.

* * *

Appendix B:

Equilibrium Accumulation Trajectory in a Two-Sector Economy

1. Harmonious Extended Reproduction Trajectory

Normal utilization of capital:

(B1) \( X_1' = \frac{K_1'}{a_1}, \quad X_2' = \frac{K_2'}{a_2} \)

Equilibrium of commodity markets:

(B2)  
\[
X_1' = a_1X_1' + a_2X_2' + I_1' + I_2' \\
X_2' = R_e(N_1' + N_2') + C_1' + C_2'
\]

Employment:

(B3) \( N_1' = \tau_1X_1' \quad N_2' = \tau_2X_2' \)

Capital accumulation:

(B4) \( K_1^{i+1} = K_1' + I_1' \quad K_2^{i+1} = K_2' + I_2' \)

Capitalists' investment/consumption ratio:

(B5) \( I_1' + I_2' = \mu(C_1' + C_2') \)

Real wage rate:

(B6) \( R_e = R \text{ (const.)} \)

The system ((B1)-(B6)) that consists of ten equations suffices to determine the movement of ten variables
\( \left( X'_1, X'_2, K'_1, K'_2, N'_1, N'_2, I'_1, I'_2, C'_1 + C'_2, R_i \right) \).

From (B1), (B2) and (B4), we get,

\[ (B7) \quad X'_i = a_1 X'_i + a_2 X'_2 \]

On the other hand, from (B2), (B3), (B5) and (B6), we get,

\[ \frac{X'_2}{X'_1} = \frac{(1 - R\tau_2) \mu + a_2}{R\tau_1 \mu + (1 - a_1)} (\equiv \lambda) \]

Inserting (B8) into (B7), we have the following difference equation,

\[ (B9) \quad X'_{i+1} = \frac{\lambda}{a_1 \lambda + a_2} X'_i. \]

Thus the rate of growth of the production in the capital goods sector is:

\[ (B10) \quad g = \frac{\lambda(1 - a_1) - a_2}{\lambda a_1 + a_2} = \frac{(1 - R\tau_1) \mu(1 - a_1)}{(1 - R\tau_2) \mu a_1 + R\tau_1 \mu a_2 + a_2} (\equiv g^*) \]

Here, \( \tau_2 \) is the labour that is necessary to produce a unit of consumption goods. It is calculated with the labour that is necessary to produce a unit of capital goods \((\tau_1)\) from the following system.

\[ (B11) \quad \begin{align*} t_1 &= a_1 t_1 + \tau_1 \\ t_2 &= a_2 t_1 + \tau_2 \end{align*} \]

The surplus condition in a two-sector economy is expressed as:

\[ (B12) \quad 1 - R\tau_2 > 0 \]

Or if we denote the rate of exploitation as \( e \),

\[ (B13) \quad e = \frac{1 - R\tau_2}{R\tau_2} > 0 \]

There are a number of features in the harmonious extended reproduction trajectory.

- The ratio of production of capital goods to that of consumption goods \( \frac{X'_1}{X'_2} \) must be constant \((\lambda)\) over time.
- The production capacity and level of production in both sectors must grow at the same rate \((g^*)\).
- The ratio of production of capital goods to that of consumption goods must be higher, as capitalists' investment/consumption ratio is higher and/or the real wage rate is smaller if there is no technical progress.
The harmonic extended reproduction function is described by the following system.

First, we have to define the harmonic extended reproduction function with technical progress. In the

3. Technological Change and the Equilibrium Accumulation function

\[ u = \frac{c + \frac{t}{n} + \frac{t}{n} + \frac{t}{n} + \frac{t}{n} + \frac{t}{n}}{n + (1 - \frac{t}{n})} = (f) \cdot \tau \]  

Relation must hold
economy can move along the harmonic extended reproduction function persistently. Thus, the following

as in the case of one-sector economy, the growth rate of labour supply must be equal to \( \theta \), so that the

\[ \frac{(n + 1)^{\theta} - \tau}{(n + 1) - \tau} \]  

The labour supply is assumed to grow at the constant rate, that is, the production (\( \theta \)). The growth rate of employment must be the same as the growth rate of production.

2. Equilibrium Accumulation function

The investment demand must grow at the same rate (\( \theta \)).

\[ \frac{c}{n} + \frac{t}{n} + \frac{t}{n} + \frac{t}{n} + \frac{t}{n} + \frac{t}{n} \]  

\[ \frac{(n + 1)^{\theta} - \tau}{(n + 1) - \tau} = (f) \cdot \tau \]

value

1. If the rate of exploitation is positive and the investment demand is not zero, the growth rate of
(B18) \( N_1' = \tau_1 X_1' \quad N_2' = \tau_2 X_2' \)

(B19) \( K_1' \tau_1 = K_1' + l_1' \quad K_2' \tau_2 = K_2' + l_2' \)

(B20) \( l_1' + l_2' = \mu (C_1' + C_2') \quad \mu: \text{const.} \)

(B21) \( e = \frac{1 - R' \tau_1'}{R' \tau_2'} (= \text{const.}) \)

(B22) \( t_1' = a_1 t_1' + \tau_1' \quad t_2' = a_2 t_2' + \tau_2' \)

(B23) \( \tau_1' = (1 - \alpha) \tau_1' \quad \tau_2' = (1 - \alpha) \tau_2' \)

As for the technical progress, we assume that the direct labour necessary to produce a unit of product in each sector decreases at the same rate. This means that the so-called organic composition of capital remain constant. The organic composition of capital (C/V by Marx's notation) can be expressed in our terminology as:

\[
\frac{C_1}{V_1} = \frac{a_1 X_1' t_1'}{R' \tau_1' X_1' t_1'} = \frac{a_1 t_1'}{\tau_1' \cdot \frac{1}{R' \tau_1'}} = \frac{a_1}{1 - a_1} \cdot \frac{1}{R' \tau_1'}
\]

\[
\frac{C_2}{V_2} = \frac{a_2 X_2' t_2'}{R' \tau_2' X_2' t_2'} = \frac{a_2 t_2'}{\tau_2' \cdot \frac{1}{R' \tau_2'}} = \frac{a_2}{1 - a_1} \cdot \frac{\tau_0}{R' \tau_2'}
\]

The term \( R' \tau_i' \) is constant, for the rate of exploitation is assumed to be constant. Thus if we specify the type of technical progress as (B22), the organic composition of capital in both sectors remain constant.

From (B17), (B18) and (B20), we obtain,

(B25) \( \lambda' = \frac{X_1'}{X_2'} = \frac{(1 - R' \tau_1') \mu + a_2}{R' \tau_1' \mu + 1 - a_1} \)

From the assumption (B21), we know \( R' \tau_2' = \text{const.} \). Using (B22), we can solve for \( t_2' \).

(B26) \( t_2' = \left( \frac{a_2}{1 - a_1} + \frac{\tau_2'}{\tau_1'} \right) \cdot \tau_1' = \left( \frac{a_2}{1 - a_1} \cdot \frac{\tau_1'}{\tau_2'} \right) \cdot \tau_2' \)

The rate of change of \( \tau_1' \) and \( \tau_2' \) is the same by assumption, so it is equal to the rate of change \( t_2' \). Then if \( R' \tau_2' = \text{const.} \) holds, then \( R' \tau_1' = \text{const.} \) and \( R' \tau_2' = \text{const.} \). Accordingly the ratio \( \lambda \) must be constant.

The growth rate of production in both sectors must be the same in order the ratio (\( \lambda \)) remain constant. We have the following difference equation.
(B27) \[ X_1^{t+1} = \frac{\lambda}{a_1 \lambda + a_2} X_1^t \]

Thus the rate of growth of the production in both sectors must be equal.

(B28) \[ g = \frac{\lambda(1 - a_1) - a_2}{\lambda a_1 + a_2} = \frac{(1 - R^t) \mu(1 - a_1)}{(1 - R^t) \mu a_1 + R^t \mu a_2 + a_2} \quad (\equiv g^*) \]

As shown above, if the rate of exploitation is constant, then the following three relations hold: \[ R^t \tau_i = \text{const.}, \quad R^t \tau_1 = \text{const.} \quad \text{and} \quad R^t \tau_2 = \text{const.} \]. Therefore \[ g = g^* (\text{: const.}) \]. From (B18), the rate of change of total labour demand is given by:

(B30) \[ 1 + d = \frac{\tau_1^{t+1} X_1^{t+1} + \tau_2^{t+1} X_2^{t+1}}{\tau_1 X_1^t + \tau_2 X_2^t} = \frac{(1 - \alpha)(1 + g^*)(\tau_1^t X_1^t + \tau_2^t X_2^t)}{\tau_1 X_1^t + \tau_2 X_2^t} = (1 - \alpha)(1 + g^*) \]

The rate on unemployment must remain constant in order to move along the harmonious extended reproduction trajectory persistently. If the rate of labour supply increases at the rate of \[ n \];

(B31) \[ L^{t+1} = (1 + n)L' \]

The relation must be satisfied.

(B32) \[ 1 + n = (1 - \alpha)(1 + g^*) \quad \therefore \quad g^* = \frac{n + \alpha}{1 - \alpha} \]

Here we assume that the rate of change of \( \tau_i \) is the same. But in the case they differ, we can demonstrate that no equilibrium accumulation trajectory exists.

In the case of

(B33) \[ \tau_1^{t+1} = (1 - \alpha_1) \tau_1^t \quad \tau_2^{t+1} = (1 - \alpha_2) \tau_2^t \quad \alpha_1 > \alpha_2, \]

the average organic composition of capital as a society falls. The average of those of two sectors rises as the organic composition of capital in capital goods sector remains constant and that of consumption goods sector fall.

(B34) \[ \frac{a_1^t}{\tau_1^t} = \frac{a_1}{\tau_1} \quad \frac{\tau_1^t}{1 - a_1} = \frac{\tau_1}{1 - a_1} = \text{const.} \]

(B35) \[ \frac{a_2^t}{\tau_2^t} = \frac{a_2}{\tau_2} \quad \frac{\tau_2^t}{1 - a_2} = \frac{a_2}{1 - a_2} \quad \frac{\tau_2}{\tau_2^0} \left( \frac{1 - \alpha_1}{1 - \alpha_2} \right)^{t} \]

Under such circumstance we cannot have an equilibrium accumulation trajectory that keeps the rate of unemployment to be constant. To keep the rate of unemployment \( (u) \) constant, the equation:

(B36) \[ (1 - u)L' = \tau_1^t X_1^t + \tau_2^t X_2^t \]

must be satisfied. Considering (B31), this can be rewritten as:

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In a capitalistic economy, the type of technical progress is decisive for the stable development of the society. Therefore, the average amount of capital in the society must be constant on average in the long run. The argument above means that technical progress cannot be maintained in the long run with or without added capital, which is shown in Figure 7. In any way, it is clear that the system cannot continue forever.

As easily demonstrated, if \( \phi < 0 \) and \( \delta' < 0 \) and \( \phi(0) = \phi_0 > 0 \), then by assumption \( \phi \) will tend to fall. The ratio will exceed the critical level of zero.

\[
\frac{\phi + \alpha - \frac{u + 1}{1 + \beta - 1}}{\phi} = \alpha (B40)
\]

When \( \alpha > 1 \) or \( \alpha < 1 \), the value of \( \phi \) will decrease.

When \( \alpha = 1 \), the value of \( \phi \) will remain constant.

If we denote \( \alpha = 1 \), the relation between \( \phi \) and \( \phi' \) equals the RHS of (B39), which can be seen in Figure 7. This result is derived from the assumption that \( \phi \) is a positive function of \( \phi \).

\[
0 < \alpha - \frac{u + 1}{1 + \beta - 1}
\]

Case 2: \( \alpha > 1 \) or \( \alpha < 1 \), which implies that \( \phi \) will decrease.

Therefore, the ratio will be equal to unity and become negative sooner or later.

From the first term of LHS of this equation, it is possible by the assumption of the technical progress.

\[
(1 - \alpha) - \frac{\alpha}{\alpha^2} + \alpha^2 - \frac{u + 1}{1 + \beta - 1} = (\alpha - 1) \cdot \left( \frac{\alpha}{\alpha^2} - \frac{1}{1 + \beta - 1} \right)
\]

We can derive the equation that shows the behavior of the ratio from (B39).

\[
\frac{i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1}{i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1} = \alpha
\]

If we denote the ratio of the employment in total employment as:

\[
\frac{i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1}{i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1} = \alpha (B38)
\]

Reformulating the relation on the ratio can be rewritten as:

\[
(i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1)(u + 1) = i^1 X_{1, 2}^2 + i^0 X_{1, 2}^1 (B39)
\]
Appendix C:

The Realized Rate of profit

Aggregate Supply = domestic production + import

\[ AS = X + F \]

Aggregate Demand = replacement demand + workers' consumption demand + capitalists' consumption demand + investment demand + government expenditure + export:

\[ AD = Z + w(W - T_w) + \pi (\Pi - T_x) + I + G + E \]

The condition of equilibrium:

\[ X + F = Z + w(W - T_w) + \pi (\Pi - T_x) + I + G + E \]

Considering the following identity,

\[ X = Z + W + \Pi \]
\[ T = T_w + T_x \]

we can rewrite (C3) as the following:

\[ Z + W + \Pi + F = Z + w(W - T_w) + \pi (\Pi - T_x) + I + G + E \]
\[ (\Pi - T_x) + T_w + (W - T_w) + T_w = w(W - T_w) + \pi (\Pi - T_x) + I + G + E - F \]
\[ (1 - \pi)(\Pi - T_x) + (1 - w)(W - T_w) = I + (G - T) + (E - F) \]

If we divide the both side by the aggregate capital K, we obtain,

\[ (1 - \pi) \frac{\Pi - T_x}{K} + (1 - w) \frac{W - T_w}{K} = \frac{I}{K} + \frac{G - T}{K} + \frac{E - F}{K} \]

That is,

\[ s_{\pi} \frac{\Pi - T_x}{K} + s_w \frac{W - T_w}{K} = \frac{I}{K} + \frac{G - T}{K} + \frac{E - F}{K} \]
References


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Figure 1: The Problems of Economics

- Economic Phenomena
  - Economic Phenomena

- Economic Activities
  - Production Relations
  - Productive Forces

1. 
2. 
3. 
4. 
5. 
6. 
7.
Figure 2: The Cumulative Process of Disequilibrium
Figure 3: The Origin of a Cumulative Process

Capitalistic Production of Commodities

Anarchy of Production

Limited Consumption of Workers

A Cumulative Process of Disequilibrium
Figure 4: Business Cycle
Figure 5: Causal Relationships of Okishio's Model

[Diagram showing the causal relationships among variables such as $K_t$, $I_t$, $L_t$, $u_t$, $g_t$, $r_t$, $\delta_t$, $X_t$, $N_t$, and their interconnections in the economic model.]
Figure 6 Classical Marxist Macroeconomic Process
(Palley (1999), p.115)
Figure 7

\[ y = F(\phi) \]