The Role of Large Creditors in Creditor Coordination Games

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Abstract

Large creditors can exert a disproportionate amount of influence in determining the likelihood of debt defaults due to coordination failure. Even if fundamentals are sound, apprehension of premature foreclosure by a single large creditor may trigger preemptive actions by other creditors, and the consequent liquidation of the distressed debtor’s assets can give rise to self-fulfilling debt defaults. To examine the influence of large creditors on the coordination problem faced by creditors, we develop a model in which a large creditor and a group of small creditors independently decide, based on private signals of fundamentals, whether to foreclose on a loan. In the absence of common knowledge of fundamentals, the incidence of failure is uniquely determined. Comparative statics on the unique equilibrium provides simple characterization of the role of large creditors. Our results show that the smaller the large creditor is, the more vulnerable the debtor is to premature foreclosure. We also find that information of relatively high precision available to the large creditor reduces the probability of failure.

JEL-Classification: G33, D82, C72

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1 Introduction

Conventional wisdom holds that large creditors can exert a disproportionate amount of influence in determining the likelihood of debt defaults due to coordination failure. Even if fundamentals are sound, apprehension of premature foreclosure by a single large creditor may induce preemptive actions by other creditors, and the consequent liquidation of the distressed debtor's assets can give rise to self-fulfilling debt defaults.

Our goal in this paper is to examine the influence of large creditors in such a strategic situation formally. In focusing on the role a large creditor plays in a creditor coordination game, we employ the equilibrium selection framework based on the global games literature. Global games, pioneered by Carlsson and van Damme (1993), are games with incomplete information whose type space is determined by the players each observing a private noisy signal of the underlying state.¹ Morris and Shin (1999) applied this framework to a creditor coordination problem, and provided a set of conditions for uniqueness in equilibrium and several comparative statics results concerning coordination risk of debt.

This paper extends the analysis of Morris and Shin (1999) by introducing a large creditor. We offer a model in which a large creditor and a continuum of small creditors independently decide, based on private signals of fundamentals, whether to foreclose on a loan.² In the absence of common knowledge of fundamentals, the incidence of failure is uniquely determined. Comparative statics on the unique equilibrium provides simple characterization of the role of large creditors. Our results show that the smaller the large creditor is, the more vulnerable the debtor is to premature foreclosure. We also find that information of relatively high precision available to the large creditor reduces the probability of failure.

The remainder of the paper is organized as follows: In Section 2, we set up a creditor coordination game with one large and many small creditors. In Section 3, we derive the unique equilibrium. In Section 4, we characterize the comparative statics properties of the equilibrium in terms of the size of the large creditor's size and the precision

¹For global games, see the excellent survey by Morris and Shin (2002).
²Asymmetric global games with a large player and a continuum of small players have been studied in the context of currency attacks, by Corsetti et al. (2001), Corsetti et al. (2002), Metz (2002), and Takeda (2000). We do not know of any attempts to apply the framework to creditor coordination problems earlier than ours.
of information available to the large creditor. Section 5 contains the conclusion. All proofs are relegated to the appendix.

2 The model

The general structure of the model is as follows. Time is divided into three periods, \( t = 0, 1, \) and 2. A firm has a project maturing at period 2 to yield a return \( v \), which is uncertain at period 0. The project is debt-financed by a continuum of ex-ante identical small creditors and a large creditor. Each small creditor has an infinitesimal portion of the whole stake. The proportion of loans financed by the large creditor is \( \lambda \in (0, 1) \). The combined mass of loans financed by small creditors then amounts to \( 1 - \lambda \). The face value of repayment is \( L > 0 \). Each creditor can receive the face value at period 2 if the realized value of \( v \) is large enough to cover repayment of debt. We assume that every creditor is rational and rationality is common knowledge.

At period 1, each creditor has the chance to decide whether to continue lending until the project matures, or to stop lending and seize the collateral \( K^* \in (0, L) \). If the collateral is liquidated following the project’s failure, it has a lower liquidation value \( K_* \in [0, K^*) \). The value of the project at maturity depends on two factors: the randomly-determined underlying fundamental state \( \theta \in \mathbb{R} \), which has a uniform prior distribution,\(^3\) and a severity of disruption \( z > 0 \) caused to the project in the event of early liquidation by creditors. Denoting by \( \ell \) the proportion of creditors who stop lending at period 1, if \( \theta \) is larger than \( z\ell \), the firm remains in operation. Otherwise, the firm is forced into bankruptcy. The realized value of the project is given by

\[
v(\theta, \ell) = \begin{cases} 
V & \text{if } z\ell < \theta, \\
K_* & \text{if } z\ell \geq \theta,
\end{cases}
\]

where \( V \) is a constant greater than \( L \).

\(^3\)As Morris and Shin (2002) point out, uniform priors are well behaved, as far as we are concerned only with conditional beliefs, and can be thought of as the limiting case where the information in the prior density becomes diffuse. This assumption enables us to concentrate on the posterior beliefs of creditors conditional on their signals and to simplify the derivation of the equilibrium. For a case of more general priors, see Takeda and Takeda (2003), which extends our model and investigates the determinants of the price of defaultable debt.
By normalizing the payoffs such that \( L = 1 \) and \( K_* = 0 \), the payoffs to a creditor are given by the following matrix, where \( \kappa \equiv (K^* - K_*)/(L - K_*) \):

<table>
<thead>
<tr>
<th></th>
<th>Project succeeds (if ( z \ell &lt; \theta ))</th>
<th>Project fails (if ( z \ell \geq \theta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continue lending</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Stop lending</td>
<td>( \kappa )</td>
<td>( \kappa )</td>
</tr>
</tbody>
</table>

For simplicity, we assume that if continued lending yields the same expected payoff as stopped lending, then a creditor prefers to stop.

Although the creditors do not observe the realization of \( \theta \) until period 2, they receive private signals regarding it. The large creditor receives the noisy signal

\[
y = \theta + \tau \eta, \tag{1}
\]

where \( \tau > 0 \) is a scale factor and \( \eta \) is a random variable with mean 0, and with smooth symmetric density \( g(\cdot) \). Let \( G(\cdot) \) be the cumulative density function for \( g(\cdot) \). Similarly, a typical small creditor \( i \) receives the signal

\[
x_i = \theta + \sigma \varepsilon_i, \tag{2}
\]

where \( \sigma > 0 \) is a scale factor and \( \varepsilon_i \) is an idiosyncratic random variable with mean 0, and with smooth symmetric density \( f(\cdot) \) and cumulative density \( F(\cdot) \). \( \varepsilon_i \) is i.i.d. across creditors and each is independent of \( \eta \). Upon receiving its respective signal, a creditor infers the value of \( \theta \) and the density of signals received by the other creditors, as well as their estimates of \( \theta \). Likewise, all other creditors form their beliefs while relying on their own private information. This assumption of incomplete information is the key to deriving the unique equilibrium.

The timing of the events is as follows:

**Period 0.** The firm invests its debt-financed fund into the project.

Nature chooses \( \theta \).

**Period 1.** Private signals regarding \( \theta \) are observed by the creditors.

The creditors decide whether to continue lending or to stop lending.
Creditors who choose to stop lending receive \( \kappa \).

Period 2. The project matures and yields a return \( v \).

Creditors who have chosen to continue lending receive 1 or 0, depending on the realization of \( v \).

3 The equilibrium

Before searching for a Bayesian Nash equilibrium in the game under the incomplete information described above, we shall briefly discuss the coordination problem under complete information. Suppose that the creditors perfectly know the value of \( \theta \) before deciding whether to continue lending. In this case, we can describe their optimal strategies as follows: If \( \theta > z \), then to continue lending is optimal, regardless of what the other creditors do, since the project will succeed even if all the other creditors stop lending. On the contrary, if \( \theta \leq 0 \), then to stop lending is optimal regardless of what the other creditors do, because the project will fail even if all the other creditors continue lending.

If \( \theta \in (0, z] \), then there is a coordination problem among the creditors. Each creditor assumes that if all the other creditors continue lending, then the payoff for continued lending is 1, so that continued lending yields more than the early liquidation value \( \kappa \). On the other hand, it assumes that if they stop lending, the payoff for continued lending is \( 0 < \kappa \), so that early liquidation is optimal. Thus, under complete information, there are two pure-strategy Nash equilibria: to continue lending and to stop lending.\(^4\)

We now show that under incomplete information there is a unique dominance-solvable equilibrium in which both types of creditors follow their respective switching strategies around the critical signals \( x^* \) and \( y^* \). To derive the unique equilibrium, we first direct our attention to solving for switching strategies around the critical signals, and then proceed to show how this solution can be obtained by the iterative elimination of strictly dominated strategies.

\(^4\)This type of coordination problem among creditors is analogous to the self-fulfilling bank run problem suggested by Diamond and Dybvig (1983), and entails multiple equilibria in the complete information game where \( \theta \) is common knowledge.
First, we consider the critical value of the fundamentals $\bar{\theta}$, above which continued lending by the small creditors alone is sufficient for the project to succeed. $\bar{\theta}$ is given by

$$\bar{\theta} = z \left( 1 - (1 - \lambda) F\left(\frac{\bar{\theta} - x^*}{\sigma}\right) \right). \quad (3)$$

We then consider the critical value of the fundamentals $\bar{\theta}$, above which the project succeeds if and only if both large and small creditors continue lending:

$$\bar{\theta} = z \left( 1 - \lambda - (1 - \lambda) F\left(\frac{\bar{\theta} - x^*}{\sigma}\right) \right). \quad (4)$$

Since both $\bar{\theta}$ and $\theta$ are functions of the small creditors’ switching point $x^*$, which depends on the large creditor’s switching point $y^*$, we need to solve these two switching points simultaneously from the respective optimization problems of the creditors. First, we consider the large creditor’s problem. Given $\theta$, its optimal strategy is to continue lending if and only if its expected payoff for continued lending conditional on $y$, which is given by $G((y - \theta)/\tau)$, exceeds its payoff for stopped lending, which equals $\kappa$. Hence, the switching point $y^*$ is defined by:

$$G\left(\frac{y^* - \theta}{\tau}\right) = \kappa. \quad (5)$$

Next, we consider a small creditor’s problem. If $\theta > \bar{\theta}$, continued lending can be successful regardless of the large creditor’s actions. If $\theta \in (\bar{\theta}, \bar{\theta})$, the project succeeds if and only if the large creditor continues lending. If $\theta \leq \bar{\theta}$, the project fails even if the large creditor continues lending. Since a small creditor’s optimal strategy is to continue lending if and only if its expected payoff for continued lending conditional on $x$ exceeding its payoff for stopped lending, the switching point $x^*$ is given by:

$$\frac{1}{\sigma} \int_{\bar{\theta}}^{\bar{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) G\left(\frac{\theta - y^*}{\tau}\right) d\theta + \frac{1}{\sigma} \int_{\bar{\theta}}^{\infty} f\left(\frac{\theta - x^*}{\sigma}\right) d\theta = \kappa. \quad (6)$$

There is a unique $x^*$ that solves (6). To see this, it is helpful to change variables in the
integrals as follows:

\[ s \equiv \frac{\theta - x^*}{\sigma}, \quad \bar{\delta} \equiv \frac{\theta - x^*}{\sigma}, \quad \text{and} \quad \bar{\delta} \equiv \frac{\bar{\theta} - x^*}{\sigma}. \] (7)

Therefore (6) can be rewritten as:

\[ \int_{\bar{\delta}}^{3} f(s) G \left( \frac{\sigma}{r} (s - \bar{\delta}) - G^{-1}(\kappa) \right) \, ds + \int_{\bar{\delta}}^{\infty} f(s) \, ds - \kappa = 0. \] (8)

Note, however, that both \( \bar{\delta} \) and \( \bar{\delta} \) are strictly decreasing in \( x^* \), since

\[ \frac{d\bar{\delta}}{dx^*} = -\frac{1}{z(1 - \lambda)f(\bar{\delta}) + \sigma} < 0, \quad \text{and} \quad \frac{d\bar{\delta}}{dx^*} = -\frac{1}{z(1 - \lambda)f(\bar{\delta}) + \sigma} < 0. \]

Given that the left-hand side of (8) is strictly decreasing in both \( \bar{\delta} \) and \( \bar{\delta} \), it is strictly increasing in \( x^* \). The left-hand side of (8) is negative for sufficiently small \( x^* \), and positive for sufficiently large \( x^* \). Given that the left-hand side of (8) is continuous in \( x^* \), there is a unique solution to (8). Once \( x^* \) is established, \( y^* \) can be determined from (5).

Thus far, we have shown that there is a unique equilibrium within switching strategies. We can finish the argument by showing that the switching equilibrium identified above is the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies. The proof is relegated to the appendix. Now we have:

**Proposition 1** There is a unique, dominance-solvable equilibrium in which the large creditor uses the switching strategy around \( y^* \) and the small creditors use the switching strategy around \( x^* \).5

Proposition 1 corresponds to Lemma 1 of Morris and Shin (1999), which shows the existence of the unique switching-strategy equilibrium in a coordination game among ex-ante identical small creditors.

5The general structure of our model conforms to a class of supermodular games in which the iterative elimination of strictly dominated strategies yields a unique equilibrium as the uniquely rationalizable strategy. See Milgrom and Roberts (1990) for rationalizable strategies in the class of supermodular games. Besides, the equilibrium of our model is not a robust equilibrium to incomplete information. See Kajii and Morris (1997) for robustness of equilibria to incomplete information.
4 The role of the large creditor

Having established the uniqueness of equilibrium, we can now examine the role of the large creditor. In contrast to the model with only small creditors discussed in Morris and Shin (1999), the model with both small and large creditors cannot be solved explicitly for the equilibrium. The four equations (3), (4), (5), and (8) jointly determine the critical states \( \bar{\theta} \) and \( \tilde{\theta} \) and the switching points \( x^* \) and \( y^* \). Although it is difficult to give definitive answers to comparative statics for general parameter values, we can obtain tidy results by focusing on the limiting case where creditors have very precise information. In such a case, fundamental uncertainty of the payoff relevant state \( \theta \) becomes smaller and smaller, while strategic uncertainty of the action of others remains as large as ever, so that we can highlight the strategic impact of the large creditor.

Let us explore the limiting case where both types of creditors have arbitrarily precise information, while the precision of the large creditor's signal relative to the small creditors' tends to \( r: \sigma \to 0, \tau \to 0, \) and \( \sigma/\tau \to r. \)

In the limit, it follows from (5) that as \( \tau \to 0, y^* \) must converge to \( \bar{\theta} \), otherwise the left-hand side of (5) converges to either zero or one, rather than \( \kappa. \) Therefore, in the limit, the large creditor continues lending at states better than \( \bar{\theta} \), and stops lending at states equal to or worse than \( \bar{\theta} \). From (3) and (4), as \( \sigma \to 0 \) the same argument holds for the small creditors, so they also follow the switching strategy around \( \bar{\theta} \) as well. Consequently, in the limit we have \( x^* = y^* = \bar{\theta} \), so that the project succeeds if and only if \( \bar{\theta} > \bar{\theta} \). In other words, the switching points for both types of creditors converge to the critical state \( \bar{\theta} \), at which the project switches from failure to success in the limit.

As Corsetti et al. (2001) point out in their currency crisis model, in solving for the critical state in the limit, it is essential to distinguish the case where the large player is sufficiently large from the case where it is not. We know from (3) and (4) that it is only when \( \bar{\theta} < z\lambda \) that \( \bar{\theta} \) is larger than \( \bar{\theta} \), while \( \bar{\theta} = \bar{\theta} \) holds when \( \bar{\theta} \geq z\lambda \). We can characterize the equilibrium value of \( \bar{\theta} \) in the limit as follows:

Proposition 2 In the limit as \( \sigma \to 0, \tau \to 0, \) and \( \sigma/\tau \to r \), the critical state \( \bar{\theta} \) converges to \( z(1 - \lambda - (1 - \lambda)F(\delta)), \) where \( \delta \) falls into two cases. If \( \bar{\theta} < z\lambda \), then \( \delta \) is
the unique solution to

\[ \int_{\delta}^{\infty} f(s) G\left(r(s - \delta) - G^{-1}(\kappa)\right) ds = \kappa. \tag{9} \]

If \( \theta \geq z\lambda \), then \( \hat{\delta} \) is the unique solution to

\[ \int_{\delta}^{M} f(s) G\left(r(s - \hat{\delta}) - G^{-1}(\kappa)\right) ds + \int_{M}^{\infty} f(s) ds = \kappa, \tag{10} \]

where

\[ M = F^{-1}\left(F(\hat{\delta}) + \frac{\lambda}{1 - \lambda}\right). \]

\textbf{Proof.} See the appendix. ■

4.1 Size of the large creditor

Now let us consider how a change in the size of the large creditor affects the probability of failure.

\textbf{Proposition 3} In the limit as \( \sigma \to 0, r \to 0, \) and \( \sigma / r \to r \), the critical state \( \theta \) is strictly decreasing in \( \lambda \).

\textbf{Proof.} See the appendix. ■

In other words, a decrease in the size of the large creditor reduces the range of fundamentals where creditors continue lending and raises the probability of project failure.

In their currency crisis model, Corsetti et al. (2001) reveal that the existence of a large trader always has a non-negative effect on the incidence of attack on the currency in the limit where traders have very precise information. They do not, however, provide a definitive conclusion on the size effect of the large trader.\(^6\) In contrast, our model shows that within the class of asymmetric global games with a large player, a case

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\(^6\)To be more precise, they suggest that it is only when the size of the large trader is sufficiently large that the size effect has an unambiguously positive effect on the incidence of attack on the currency in the limiting case. When the size is small, the size effect is ambiguous in their model.
exists where we can unambiguously identify the global sign of the effect of the large player's size on the equilibrium, regardless of the large player's size itself.

4.2 Relative precision of information available to the large creditor

Next, we consider how a change in the precision of the large creditor's signal relative to the small creditors' signals influences the probability of failure.

**Proposition 4** In the limit as \( \sigma \to 0, \tau \to 0, \) and \( \sigma/\tau \to r, \) the critical state \( \theta \) is strictly decreasing in \( r \).

**Proof.** See the appendix.

In other words, a decrease in the precision of the large creditor's signal relative to the small creditors' reduces the range of fundamentals where creditors continue lending and raises the probability of project failure.

Our results show that increasing the size of the large creditor, as well as increasing the relative precision of information available to it, increases the likelihood of project success, which is at the Pareto superior equilibrium in our model. On the contrary, Corsetti et al. (2001) argue that when the large trader is sufficiently large, increasing the size of it, as well as increasing the relative precision available to it, increases the likelihood of peg failure, which is at the Pareto inferior equilibrium in their currency crisis model. The contrastive results come from the difference in the payoff structure between the two models. The large creditor in our model receives the better payoff when the project succeeds, whereas the large trader in their model receives the better payoff when the peg fails. If we construct a creditor coordination game where a large creditor can make a profit from a debtor's bankruptcy, the comparative statics will likely provide quite different results from those in this paper.

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\footnote{In a symmetric global game \( \text{à la} \) Morris and Shin (1999), the equilibrium in the limit does not depend on the structure of the noise. Proposition 4 implies that noise-independent equilibrium selection fails in our model with payoff asymmetry among creditors. See Frankel \textit{et al.} (2003) for noise-independent equilibrium selection.}
5 Concluding remarks

In this paper, we have studied a coordination game played by a large creditor and a continuum of small creditors, and examined the influence of the large creditor on the equilibrium. In the absence of common knowledge of fundamentals, the incidence of failure is uniquely determined. Results from comparative statics on the unique equilibrium show that increasing the size of the large creditor, as well as increasing the relative precision of information available to the large creditor, reduces the range of fundamentals where creditors stop lending, and tends to prevent project failure. These findings imply that, under circumstances where self-fulfilling beliefs as well as fundamentals play an important role in determining outcomes, a solvent but illiquid debtor who would be forced into bankruptcy by the creditors' disruptive early liquidation without a large creditor, may well be expected to be saved by successful creditor coordination with a large, well-informed creditor.

Appendix

Proof of Proposition 1

We complete the proof for proposition 1 by showing that the switching strategy is the only equilibrium strategy that survives iterative elimination of strictly dominated strategies. Let us consider a small creditor's best response conditional on $x$ when all other small creditors follow the switching strategy around $\bar{x}$ and the large creditor plays its best response against this switching strategy, which we know is the switching strategy around $y(\bar{x})$, by using (5). The net expected payoff for a small creditor from continued lending as opposed to stopped lending is given by

$$\pi(x, \bar{x}) = \frac{1}{\sigma} \int_{\tilde{\theta}(\bar{x})}^{\tilde{\theta}(\bar{x})} f \left( \frac{\theta - x}{\sigma} \right) G \left( \frac{\theta - y(\bar{x})}{\tau} \right) d\theta + \frac{1}{\sigma} \int_{\tilde{\theta}(\bar{x})}^{\infty} f \left( \frac{\theta - x}{\sigma} \right) d\theta - \kappa,$$

where $\tilde{\theta}(\bar{x})$ and $\tilde{\theta}(\bar{x})$ denote the values of $\tilde{\theta}$ and $\tilde{\theta}$ when small creditors follow the switching strategy around $\bar{x}$. Note that $\pi(\cdot, \cdot)$ is strictly increasing in its first argument and strictly decreasing in its second.

For sufficiently good signals, the net expected payoff for a small creditor from
continued lending is positive, and continued lending is a dominant strategy for a small creditor, regardless of what the other creditors do. Let $\bar{x}_1$ be the threshold value of $\bar{x}$, above which continued lending is a dominant strategy for a small creditor. Since every creditor is rational, it does not use a dominated strategy. Furthermore, since each creditor knows that the others are rational, it infers that the others will not use a dominated strategy either. Thus, for a small creditor to stop lending above $\bar{x}_1$ cannot be a dominant strategy. As a consequence, to stop lending cannot be a dominant strategy for a small creditor whenever its signal is above $\bar{x}_2$ such that it solves $\pi(\bar{x}_2, \bar{x}_1) = 0$. Since the switching strategy around $\bar{x}_2$ is the best response to the switching strategy around $\bar{x}_1$, even the small creditor who assumes the lowest possibility of the project’s success believes that the incidence of continued lending is higher than that implied by the switching strategy around $\bar{x}_1$ and the large creditor’s best response $y(\bar{x}_1)$. Since $\pi(\cdot, \cdot)$ is strictly increasing in its first argument, any strategy to stop lending for signals higher than $\bar{x}_2$ is strictly dominated. Proceeding in this way, we have the following decreasing sequence:

$$
\bar{x}_1 > \bar{x}_2 > \bar{x}_3 > ... > \bar{x}_k > ... ,
$$

where any strategy to stop lending for signal $x > \bar{x}_k$ does not survive $k$ steps of elimination of strictly dominated strategies. Common knowledge of rationality takes this procedure to the limit. Sequence $\{\bar{x}_k\}_{k=0}^{\infty}$ is monotone and bounded, so that the limiting point $\bar{x}_\infty = \lim_{k \to \infty} \bar{x}_k$ exists and is given by $\bar{x}_\infty = \sup(x | \pi(x, x) = 0)$. $\bar{x}_\infty$ is the largest solution to $\pi(x, x) = 0$. Any strategy to stop lending above $\bar{x}_\infty$ does not survive the iterative elimination of strictly dominated strategies.

A similar argument applies to the smallest solution to $\pi(x, x) = 0$, and thus there exists $\underline{x}_\infty$ such that $\underline{x}_\infty = \inf(x | \pi(x, x) = 0)$. Any strategy that continues lending below $\underline{x}_\infty$ does not survive iterative elimination. However, if $\pi(x, x) = 0$ has a unique solution, then the largest solution $\bar{x}_\infty$ must coincide with the smallest solution $\underline{x}_\infty$. Hence, there is only one strategy that still remains after eliminating all strictly dominated strategies, implying that this switching strategy is the unique equilibrium strategy. ■
Proof of Proposition 2

First, suppose that \( \lim \theta < z\lambda \), so that \( \lim \theta < \lim \tilde{\theta} \). Since \( x^* \to \theta \), \( (\theta - x^*)/\sigma \to \infty \) must hold, and it follows that \( \tilde{\theta} \to \infty \). Thus, (8) can be written as (9). Next, suppose that \( \lim \theta \geq z\lambda \), so that \( \lim \theta = \lim \tilde{\theta} \). Since \( x^* \to \tilde{\theta} \), \( \tilde{\theta} \) must be finite. We know from (3) and (4) that \( 1 - (1 - \lambda)F(\tilde{\theta}) = 1 - \lambda - (1 - \lambda)F(\delta) \). Thus, \( \tilde{\theta} = F^{-1}(F(\delta) + \lambda/(1 - \lambda)) \) and (8) can be written as (10). In both (9) and (10), the left-hand side is strictly increasing in \( \delta \), so that there is a unique value of \( \delta \) that solves both equations. Accordingly, the proposition follows from (4). ■

Proof of Proposition 3

When \( \theta < z\lambda \), (9) holds. The left-hand side of (9) is strictly decreasing in \( \lambda \) through its effect on \( M \), so that \( \delta \) is strictly increasing in \( \lambda \). When \( \theta < z\lambda \), (10) holds. Since the left-hand side of (9) does not depend on \( \lambda \), \( \delta \) does not depend on \( \lambda \) either. Hence, regardless of the size of \( \lambda \), \( \delta \) is non-decreasing in \( \lambda \). It follows from (4) that the overall effect of \( \lambda \) on \( \delta \) is given by

\[
\frac{d\theta}{d\lambda} = -z \left( (1 - F(\delta)) + (1 - \lambda)f(\delta) \frac{d\delta}{d\lambda} \right). \tag{11}
\]

Since \( \delta \) is non-decreasing in \( \lambda \), the left-hand side of (11) must be negative. Thus, \( \theta \) is strictly decreasing in \( \lambda \). ■

Proof of Proposition 4

In both (9) and (10), the left-hand side is strictly increasing in \( r \), while strictly decreasing in \( \delta \). Therefore, \( \delta \) must be strictly increasing in \( r \). It follows from (4) that as \( \delta \) increase, \( \theta \) must decrease. Hence, \( \theta \) is strictly decreasing in \( r \). ■

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